Polariton rings and ring- based metamaterials

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Outline

a. Magnetic field in classical dynamics
b. Magnetic field in quantum physics and AB effect
c. Optically induced AB effect in mesoscopic rings
d. Systems of interconnected rings
e. Optical AB effect for excitons
f. Polariton rings and geometric phase
g. Edge states in polariton ring lattices

Regimes of light- matter interaction

2DEG irradiated by electromagnetic field

Weak light-matter interaction	Strong light-matter interaction
Electron spectrum unperturbed	Electron spectrum modified (dynamic Stark effect)
Interact via photon emission/absorption	Interact via electron “dressing”
Effects: photovoltaic, high-frequency conductivity	Effects: exciton-polaritons in microcavities; band gap opening and metal-insulator transition in graphene; artificial U(1) gauge fields

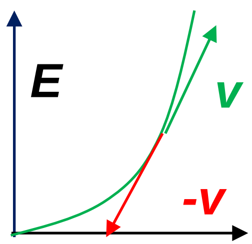
Magnetic field in classical and quantum physics

Classical physics

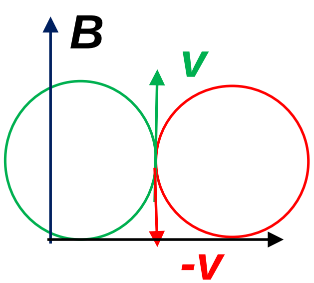
Lorentz Force

$$\frac{d^2\boldsymbol{r}}{dt^2} = \boldsymbol{F} = q\boldsymbol{E} + q[\boldsymbol{v} \times \boldsymbol{B}]$$

Time inversion symmetry



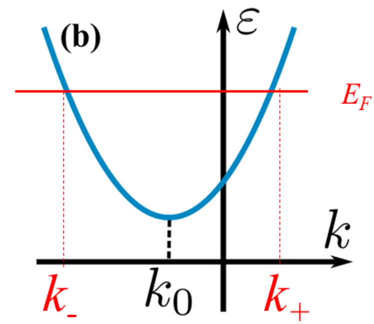
$$t \to -t$$
$$E \to E$$



$$t \to -t$$
$$B \to -B$$

Quantum physics

$$\hat{H}_0 = \frac{1}{2m_e}\left(\hat{p}_\varphi - eA_\varphi\right)^2 \quad \hat{p}_\varphi = i\hbar\frac{\partial}{\partial\varphi} \quad A_\varphi = \frac{\Phi}{2\pi R}$$

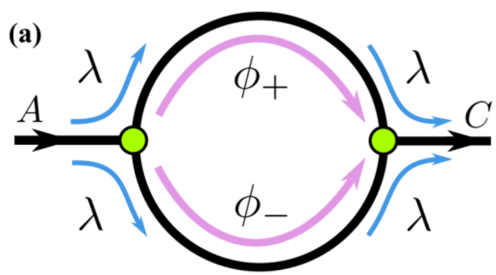


$$\Psi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \qquad k = m/R$$

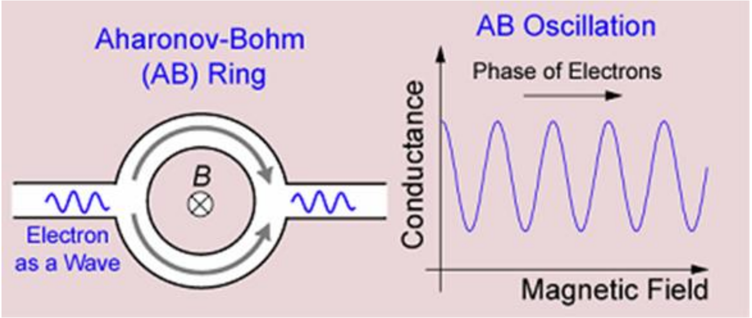
$$\varepsilon(m) = \frac{\hbar^2}{2m_eR^2}\left(m + \frac{\Phi}{\Phi_0}\right)^2 \quad \varepsilon(k) = \frac{\hbar^2}{2m_e}\left(k + \frac{\Phi}{R\Phi_0}\right)^2$$

Time reversal asymmetry

$$E(k) \neq E(-k) \Rightarrow \Delta\phi = \pi R(k_+ - k_-) \neq 0$$



$$G = 2\frac{e^2}{h}\left(1 - \left|\frac{\sin^2\left(\frac{\Phi}{2\Phi_0}\right)}{1 - \exp(2\pi i k_F R)\cos^2\left(\frac{\Phi}{2\Phi_0}\right)}\right|^2\right)$$

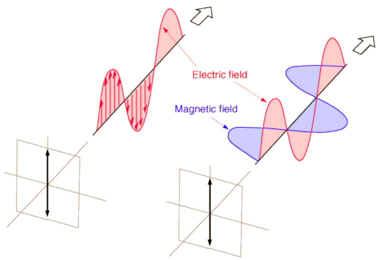


Aharonov- Bohm oscillations in conductance

Time- reversal symmetry breaking by circular polarized light

AB effect is a consequence of non-equivalency of clockwise and anticlockwise motion, i.e. of the breaking of time- reversion symmetry

$$\hat{H}_0 = \frac{1}{2m_e}\left(\hat{p}_\varphi - eA_\varphi\right)^2 \qquad \hat{\mathcal{H}} = \hat{\mathcal{H}}_R^{(0)} + \hat{U}_R$$

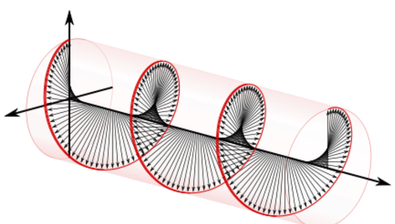


Linear polarized light: time inversion does not change polarization

$$\hat{U}_R = -ieR\sqrt{\frac{\pi\hbar\omega}{V}}\left(e^{i\varphi}\hat{a} - e^{-i\varphi}\hat{a}^\dagger\right)$$

$$\varepsilon_{m,N} = \varepsilon^{(0)}_{m,N} + \frac{|\langle m+1, N-1|\hat{U}_R|m,N\rangle|^2}{\varepsilon^{(0)}_{m,N} - \varepsilon^{(0)}_{m+1,N-1}} + \frac{|\langle m-1, N+1|\hat{U}_R|m,N\rangle|^2}{\varepsilon^{(0)}_{m,N} - \varepsilon^{(0)}_{m-1,N+1}}.$$

$$\varepsilon(m) = m^2\varepsilon_R + eE_0R\left[\frac{eE_0R/2\varepsilon_R}{(2m - \hbar\omega/\varepsilon_R)^2 - 1}\right]$$

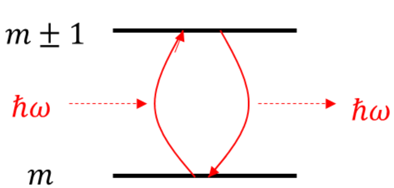


Circular polarized light: time inversion changes polarization

$$\varepsilon(k) = \frac{\hbar^2k^2}{2me} + \frac{\hbar e^2{E_0}^2}{2me^2R\omega^3}k = \frac{\hbar^2}{2me}\left(k + \frac{{E_0}^2}{2me\hbar 2R\omega^3}\right)^2 + const$$

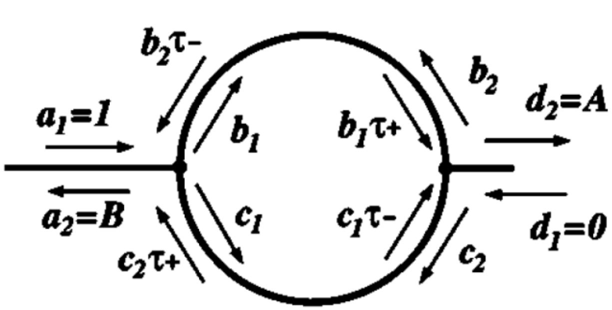
$$\varepsilon(k) = \frac{\hbar^2}{2me}\left(k + \frac{\Phi}{R\Phi_0}\right)^2$$

Illumination of the ring by circular polarized light leads to the appearance of an effective magnetic flux and syntetic U(1) gauge potential



$$\Phi_{eff} = \frac{e\pi{E_0}^2}{2me\omega^3} \qquad A_{eff} = \frac{e{E_0}^2}{4me\omega^3R}$$

Optical AB oscillations



$$\begin{pmatrix} b_1 \\ a_2 \\ c_1 \end{pmatrix} = \begin{pmatrix} r & \varepsilon & t \\ \varepsilon & \sigma & \varepsilon \\ t & \varepsilon & r \end{pmatrix} \cdot \begin{pmatrix} b_2\tau_- \\ 1 \\ c_2\tau_+ \end{pmatrix}$$

$$\begin{pmatrix} b_2 \\ d_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} r & \varepsilon & t \\ \varepsilon & \sigma & \varepsilon \\ t & \varepsilon & r \end{pmatrix} \cdot \begin{pmatrix} b_1\tau_+ \\ 0 \\ c_1\tau_- \end{pmatrix}$$

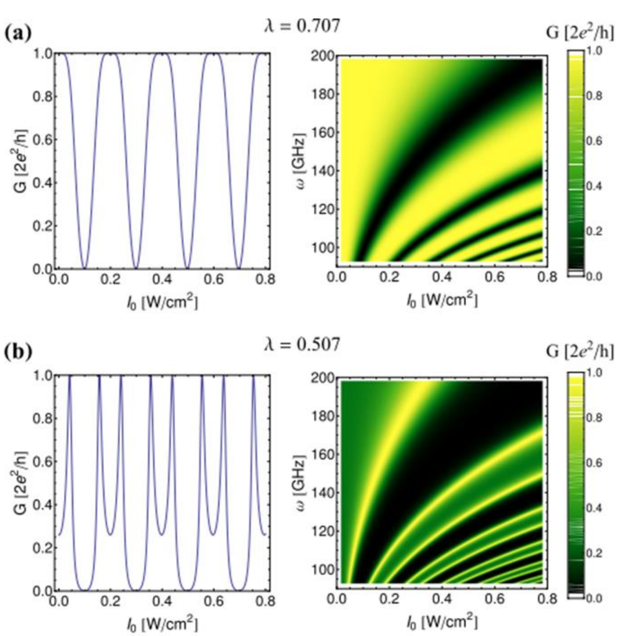
$$\tau_\pm = e^{i\pi k_\pm R}$$

$$k_\pm = k \pm \frac{\Phi_{eff}}{2\Phi_0}$$

$$G = \frac{2e^2}{h}|A|^2$$

For fully transparent contacts (σ=0):

$$G = \frac{2e^2}{h}\left[1 - \left|\frac{\sin^2(\Phi_{\rm eff}/2\Phi_0)}{1-\exp(i2\pi R k_F)\cos^2(\Phi_{\rm eff}/2\Phi_0)}\right|^2\right]$$



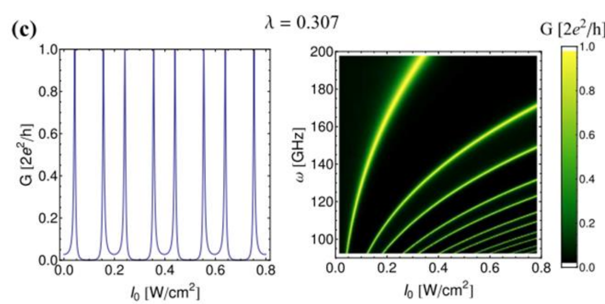
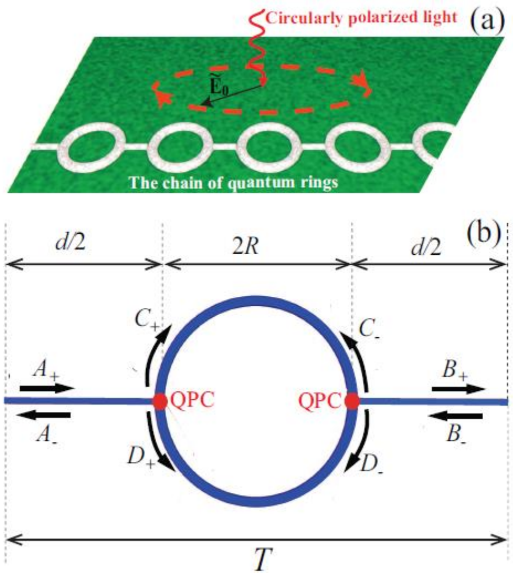


FIG. 2: (Color online) Conductance of a mesoscopic ring, G, under a circularly polarized electromagnetic wave as a function of wave intensity I_0 and wave frequency ω. Plots (a), (b) and (c) correspond to different transmission amplitudes λ between the current leads and the ring. Frames in the left column are fixed at the wave frequency ω = 100 GHz. In all plots, the ring parameters are assumed to be R = 10 μm, ε_F = 10 meV, and m_e = 0.1 m_{e0}, where m_{e0} is the mass of free electron.

1D array of rings

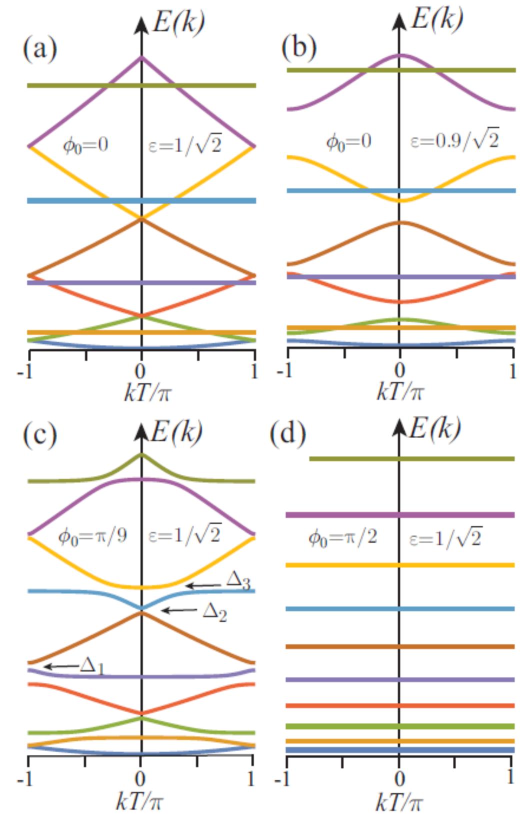


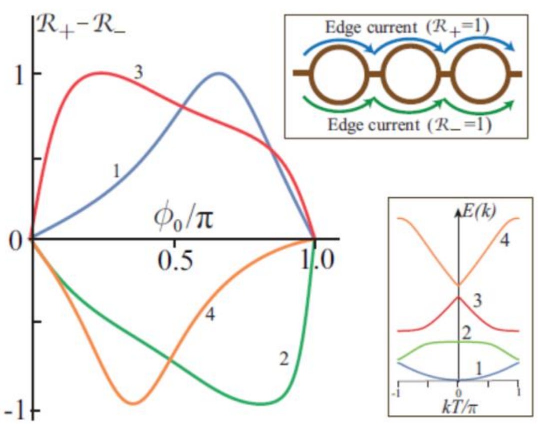
$$\begin{pmatrix} A_+ \\ A_- \end{pmatrix} = e^{ikT} \begin{pmatrix} B_+ \\ B_- \end{pmatrix}$$

$$\begin{pmatrix} A_- e^{-iqd/2} \\ C_+ \\ D_+ \end{pmatrix} = S \begin{pmatrix} A_+ e^{iqd/2} \\ C_- e^{i(\pi q R - \phi_0)} \\ D_- e^{i(\pi q R + \phi_0)} \end{pmatrix},$$

$$\begin{pmatrix} B_+ e^{-iqd/2} \\ C_- \\ D_- \end{pmatrix} = S \begin{pmatrix} B_- e^{iqd/2} \\ C_+ e^{i(\pi q R + \phi_0)} \\ D_+ e^{i(\pi q R - \phi_0)} \end{pmatrix},$$

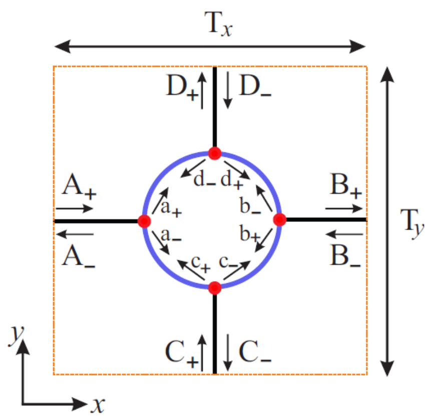
$$S = \begin{pmatrix} \sqrt{1-2\varepsilon^2} & \varepsilon & \varepsilon \\ \varepsilon & \frac{-(1+\sqrt{1-2\varepsilon^2})}{2} & \frac{(1-\sqrt{1-2\varepsilon^2})}{2} \\ \varepsilon & \frac{(1-\sqrt{1-2\varepsilon^2})}{2} & \frac{-(1+\sqrt{1-2\varepsilon^2})}{2} \end{pmatrix}$$

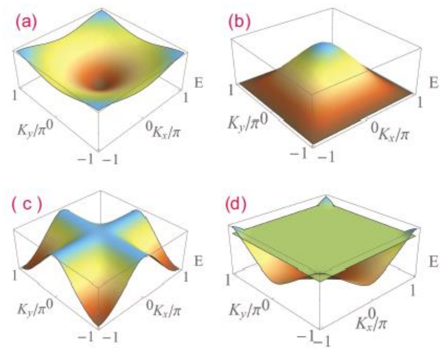


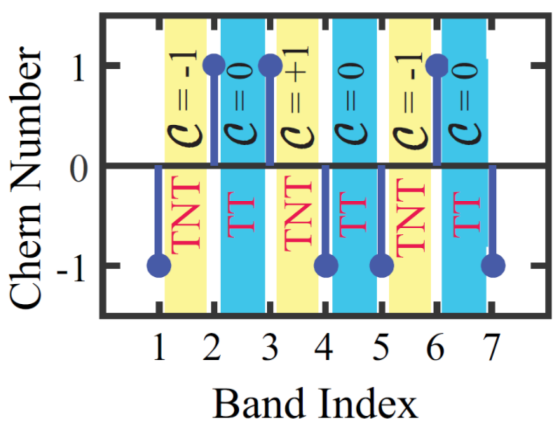


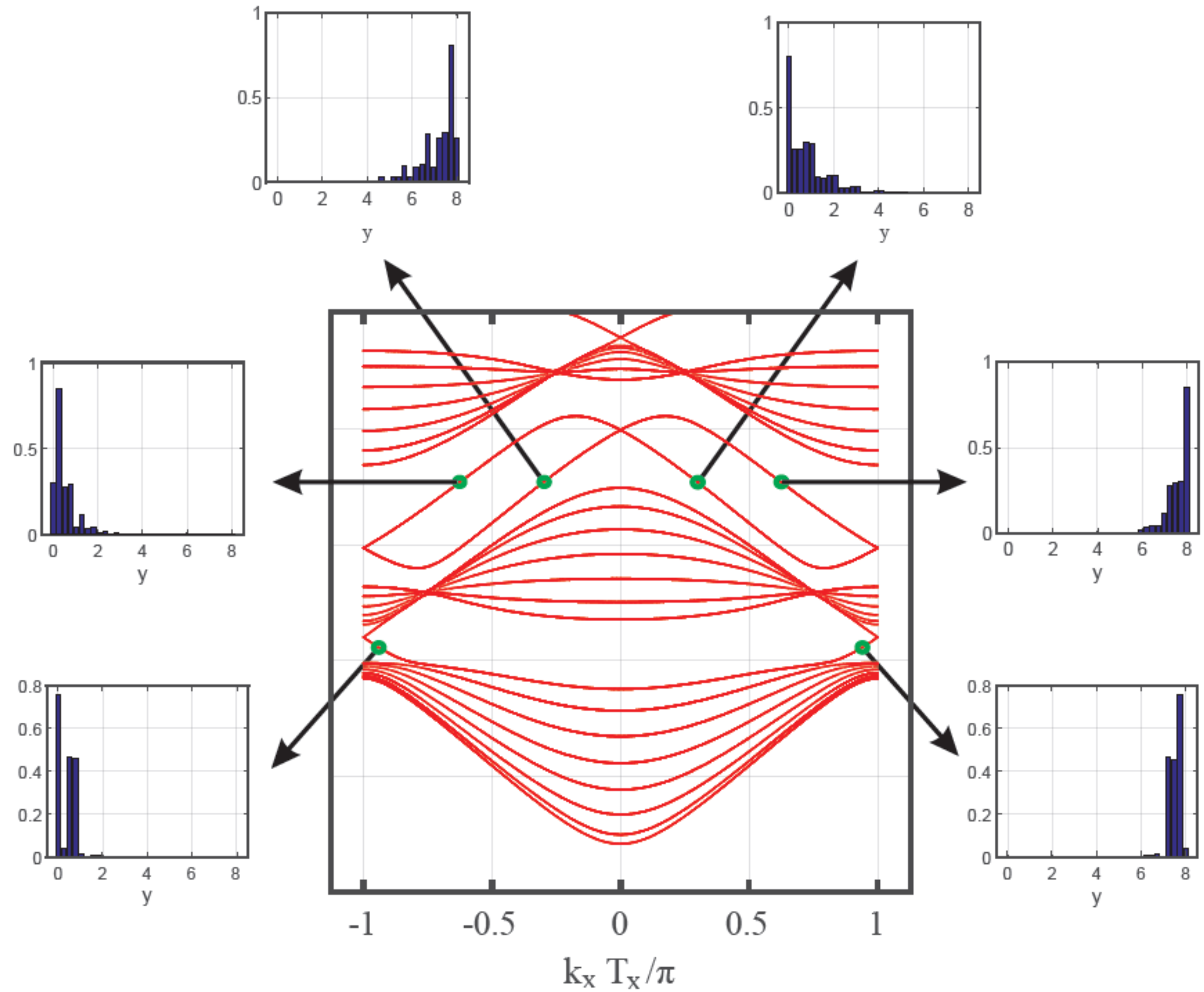
$$\sin(qd)\Big[(1-\varepsilon^2)\cos^2\phi_0 + \sqrt{1-2\varepsilon^2}\sin^2\phi_0 - \cos(2\pi q R)\Big] + \varepsilon^2\Big[\sin[q(d-2\pi R)] + 2\cos\phi_0 \sin(\pi q R)\cos(kT)\Big] = 0,$$

2D array of rings



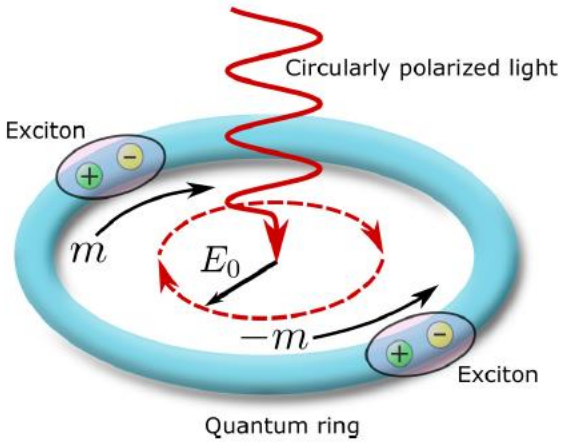






$$C = \sum_n c_n$$

Optical AB effect for excitons



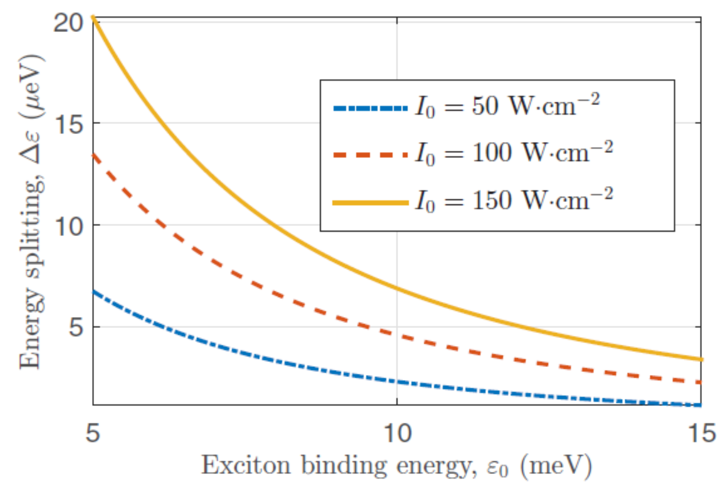
$$\hat{U} = \frac{i q_e R}{2}\sqrt{\frac{\hbar\omega}{\epsilon_0 V_0}}[(e^{-i\varphi_e} - e^{-i\varphi_h})\hat{a}^\dagger + (e^{i\varphi_h} - e^{i\varphi_e})\hat{a}]$$

$$\Delta\tilde{\varepsilon}_{0,m} = \frac{\hbar\omega}{2}\left(\frac{m_h - m_e}{M}\right)^2 (eE_0 a)^2 \left[\frac{1}{\varepsilon_R^2(1-2m)^2 - (\hbar\omega)^2} - \frac{1}{\varepsilon_R^2(1+2m)^2 - (\hbar\omega)^2}\right]. \quad (12)$$

Exciton Bohr radius

$$\Phi_{eff} = \frac{e\pi {E_0}^2}{2me\omega^3}$$

Effective flux depends on mass! And masses of an electron and a hole are different, so there appears uncompensated flux for the exciton!



Geometric Berry Phase

$$H(x(t))|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$H(x(t))|n(x(t))\rangle = E_n(x(t))|n(x(t))\rangle$$

Adiabatic approximation:

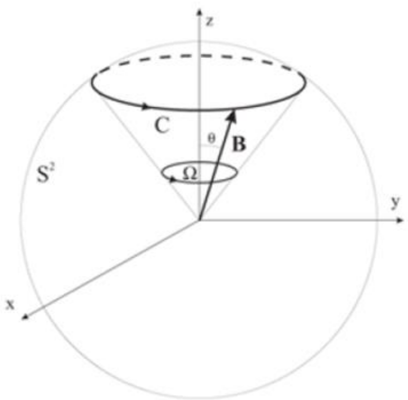
$$|\psi(t)\rangle = e^{i\phi_n}|n(x(t))\rangle$$

$$\phi_n = \theta_n + \gamma_n$$

Dynamic phase

$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(\tau)d\tau$$

Geometric phase: adiabatic evolution of spin in slowly changing time dependent magnetic field



$$\vec{B}(t) = B_0\begin{pmatrix}\sin\theta\cos(\omega t)\\ \sin\theta\sin(\omega t)\\ \cos\theta\end{pmatrix}$$

$$|n_+(t)\rangle = \begin{pmatrix}\cos\frac{\theta}{2}\\ e^{i\omega t}\sin\frac{\theta}{2}\end{pmatrix}, \quad |n_-(t)\rangle = \begin{pmatrix}-\sin\frac{\theta}{2}\\ e^{i\omega t}\cos\frac{\theta}{2}\end{pmatrix}$$

Geometric phase

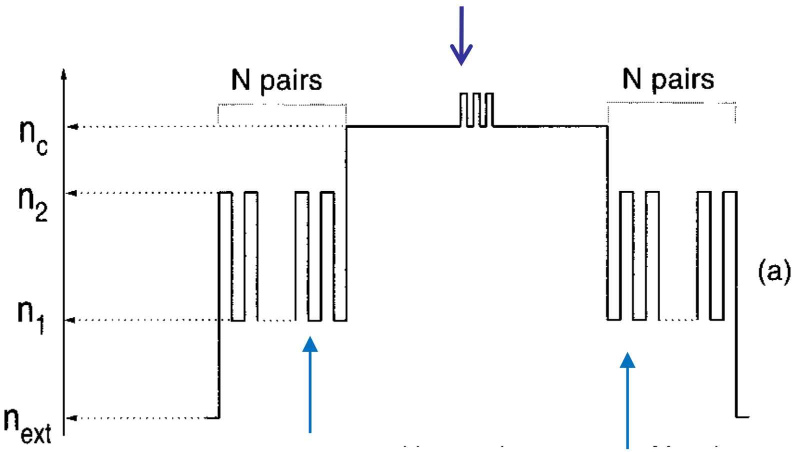
$$\gamma_n(C) = i\oint_C \langle n(x)|\nabla|n(x)\rangle dx$$

$$\gamma_\pm(C) = -\pi(1\mp\cos\theta)$$

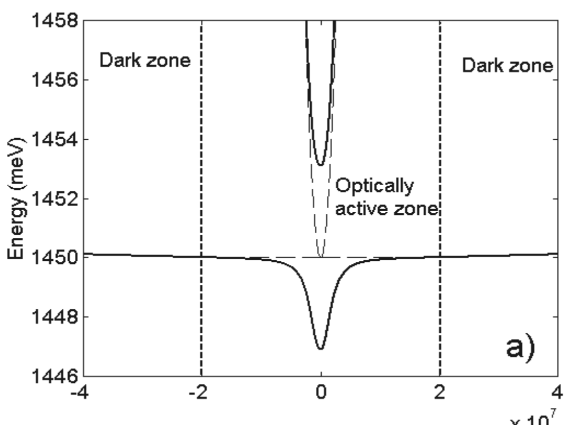
$$\gamma_\pm(C) = \mp\frac{1}{2}\Omega(C)$$

Quantum microcavities

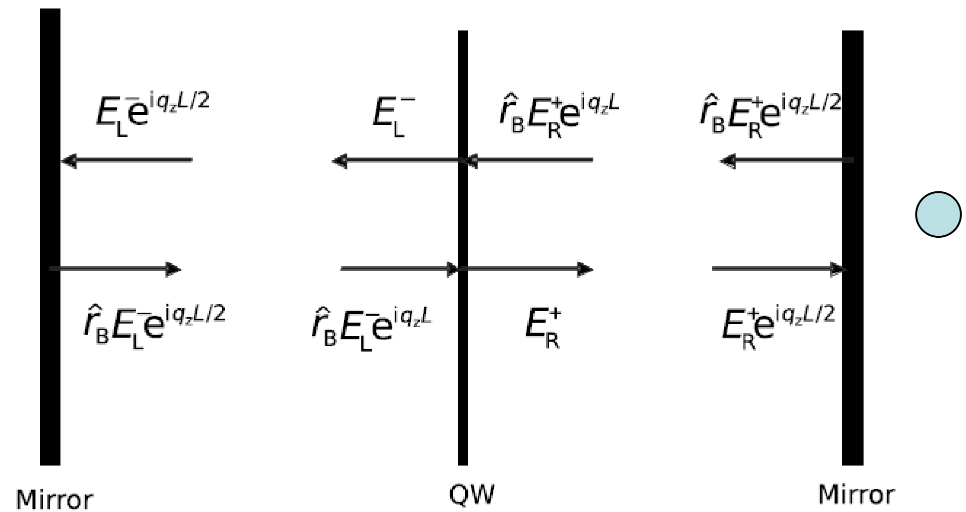
QW



Bragg mirrors



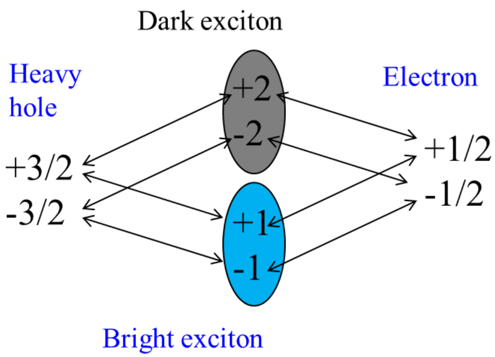
The role of the Bragg mirrors: confinement of electromagnetic field. For infinite number of the layers the photonic bandgap is formed in the mirrors, and photon becomes perfectly confined inside a cavity.



Confined photon aquires a mass

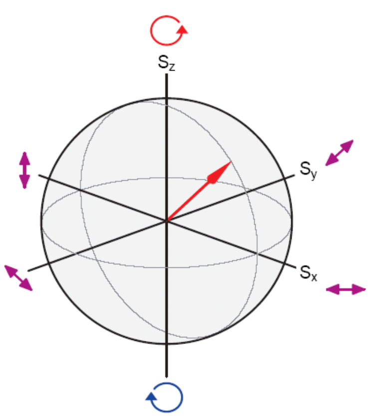
$$\hbar\omega_c = \hbar\frac{c}{n_c}\sqrt{q^2+k_z^2} \approx \hbar\frac{c}{n_c}k_z\left(1+\frac{q^2}{2k_z^2}\right) \equiv \frac{hc}{n_cL_c}+\frac{\hbar^2q^2}{2m_{ph}} \qquad m_{ph} = \frac{hn_c}{cL_c} \sim 10^{-4} \div 10^{-5}\, m_e$$

Spin and polarization of cavity polaritons



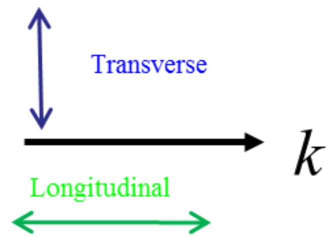
Density matrix of a two level system

$$\rho_{\vec{k}} = \frac{N_{\vec{k}}}{2} I + \vec{S}_{\vec{k}} . \sigma = \begin{pmatrix} N_{\vec{k}}/2 + S_{\vec{k},z} & S_{\vec{k},x} - iS_{\vec{k},y} \\ S_{\vec{k},x} + iS_{\vec{k},y} & N_{\vec{k}}/2 - S_{\vec{k},z} \end{pmatrix}$$



$$S_z = \frac{1}{2}\left(N_{\uparrow} - N_{\downarrow}\right)$$

Polariton TE- TM splitting

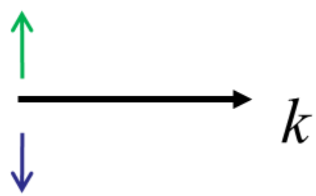


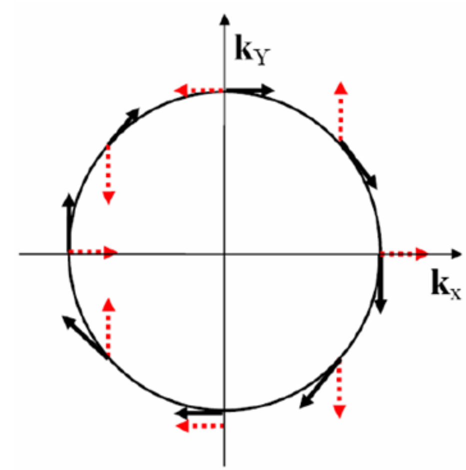
$$H = \begin{pmatrix} H_0(k) & \beta(k_x - ik_y) \\ \beta(k_x + ik_y) & H_0(k) \end{pmatrix} = H_0(k)\mathbf{I} + \mathbf{\Omega}_{LT} \cdot \boldsymbol{\sigma}$$

$$\mathbf{\Omega}_{LT} = |\mathbf{\Omega}_{LT}|(\mathbf{e}_x \sin 2\varphi + \mathbf{e}_y \cos 2\varphi)$$

$$\vec{\Omega}_{LT} = \vec{\Omega}_{LT,ex}\left|X_E^2\right| + \vec{\Omega}_{LT,ph}\left|X_P^2\right|$$

Analog for electrons: Rashba splitting

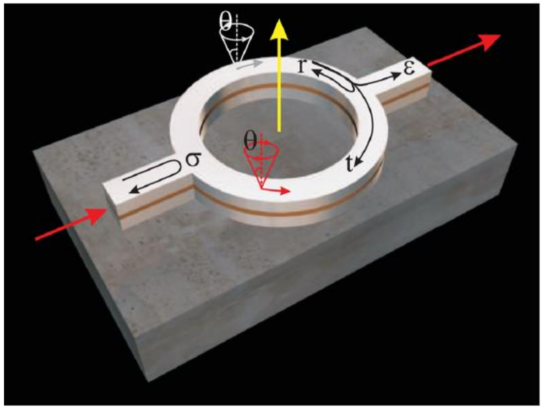


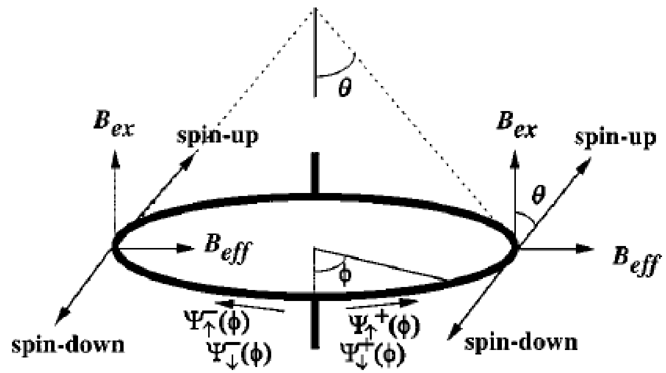


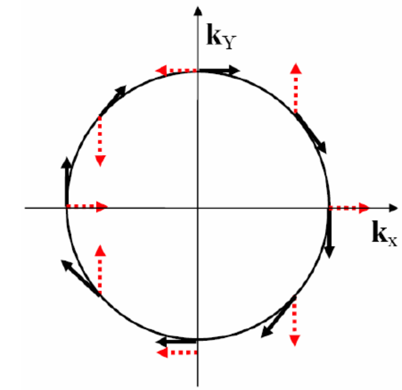
$$H = \begin{pmatrix} H_0(k) & \alpha(k_x - ik_y) \\ \alpha(k_x + ik_y) & H_0(k) \end{pmatrix} = H_0(k)\mathbf{I} + \mathbf{\Omega}_R \cdot \boldsymbol{\sigma}$$

$$\mathbf{\Omega}_R = |\mathbf{\Omega}_R|(\mathbf{e}_x \sin\varphi + \mathbf{e}_y \cos\varphi)$$

Optical analog of AB interferometer







$$H = \begin{pmatrix} -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\varphi^2} - \frac{\Delta_Z}{2} & \frac{1}{2}\Delta_{LT}e^{2i\varphi} \\ \frac{1}{2}\Delta_{LT}e^{-2i\varphi} & -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\Delta_Z}{2} \end{pmatrix}$$

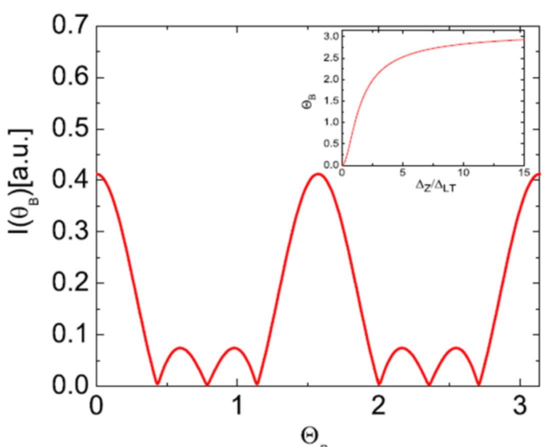
TE-TM splitting

Zeeman splitting

$$\Psi_+(\phi) = \frac{1}{\sqrt{1+\xi^2}}\begin{pmatrix} -\xi e^{+i\phi} \\ e^{-i\phi} \end{pmatrix} e^{ik_+ a\phi}$$

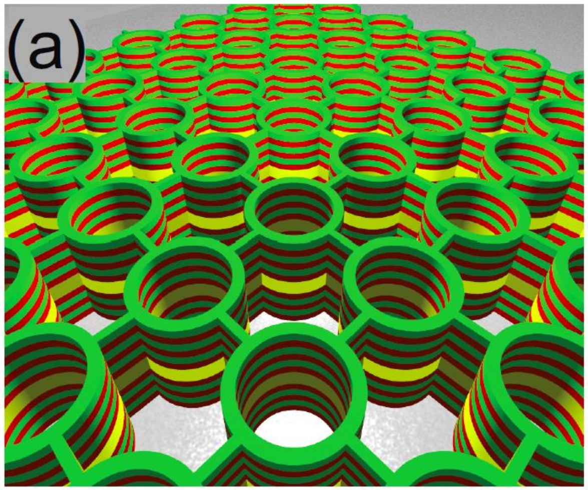
$$\Psi_-(\phi) = \frac{1}{\sqrt{1+\xi^2}}\begin{pmatrix} e^{+i\phi} \\ \xi e^{-i\phi} \end{pmatrix} e^{ik_- a\phi}$$

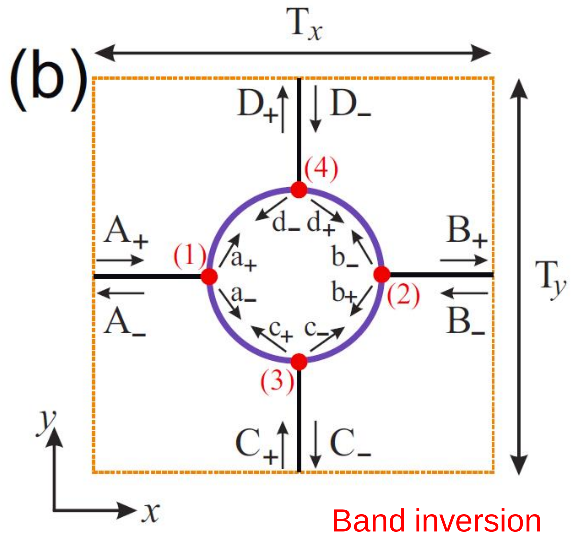
$$\xi = \frac{\Delta_{LT}/2\Delta_Z}{1+\sqrt{(\Delta_{LT}/2\Delta_Z)^2+1}}$$

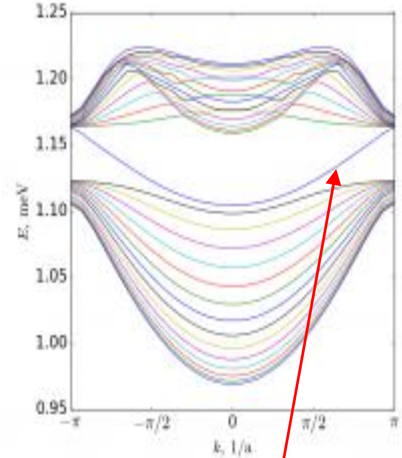


$$\theta_B = \pm\pi\left(1 - \frac{\Delta_Z}{\sqrt{\Delta_Z^2 + \Delta_{LT}^2}}\right)$$

2D Arrays of polariton rings



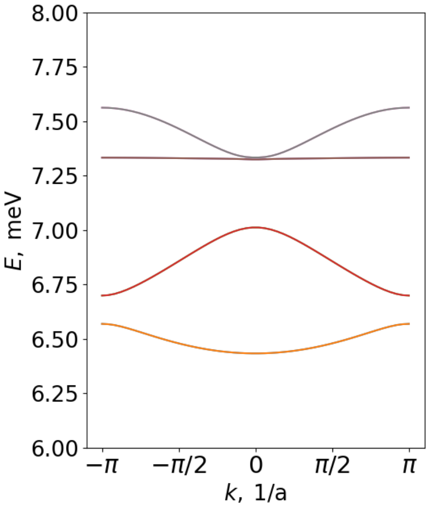




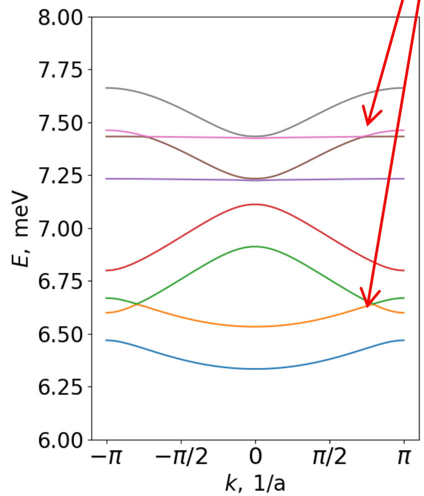
Band inversion

Bandgap openings: formation of topological edge modes

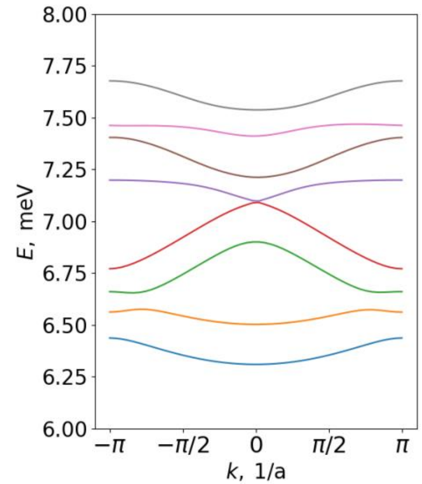
No magnetic fields



Zeeman only



Zeeman+ TE-TM



Thank you for attention

Спасибо за внимание

Ég þakka ykkur fyrir áheyrnina