Proposal of universal schemes of (classical) computaions based on Cellular Automata – a proposal for optical computation

By

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Outline

- Cellular Automata
- Optical information processing
- Exciton-polariton
- Cellular Automata in arrays of 2D Cavities

Invention of Cellular Automata

- Cellular Automata were introduced in the late 1940´s by John von Neumann and Stanislaw Ulam.
- In the late 1970´s, John Horton Conway developed the "Game of Life".
- In the 1980s, Stephen Wolfram studied of one-dimensional cellular automata, calls elementary cellular automata.
- "A New Kind of Science" in 2002.

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What is Cellular Automata?

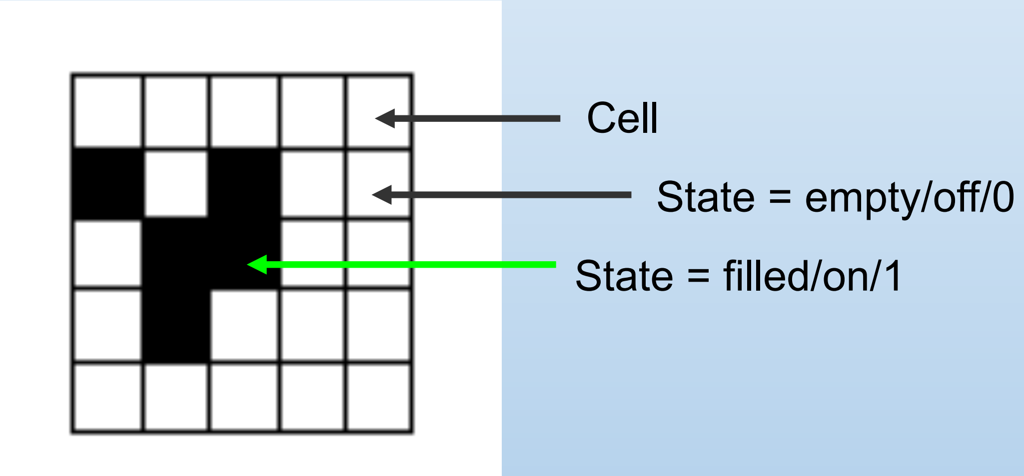
- A cellular automaton (CA) is a discrete model studied in computability theory, mathematics, physics, complexity science, theoretical biology and microstructure modelling.

- It consists of a regular grid of cells.

- Cell ------ states
  ------ neighborhood

- Each is in either off or on (0 or 1).

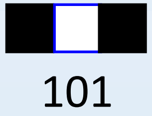
- These automata are discrete dynamic systems



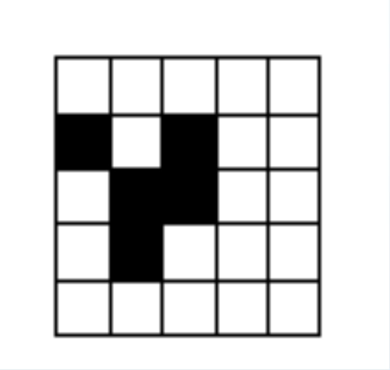
Cellular Automata

- The cells can be:
  - 1-D (i.e. just a line of cells)
  - 2-dimensional squares,
  - 3-dimensional blocks
  - There is no theoretical limit to the number of dimensions

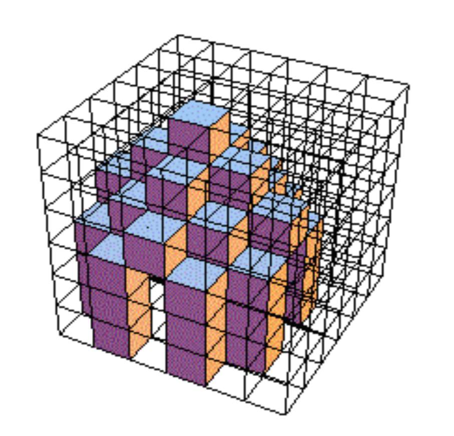
1-D



2-D



3-D



- Each cell has a defined neighborhood.

- In 1-D, every cell has a left and a right neighbor

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Two-dimensional Automaton

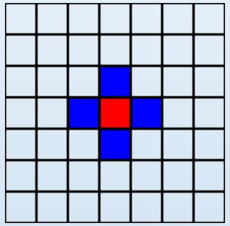
- In 2D , the two most common types of neighbourhood

(i) Von Neumann neighborhood:

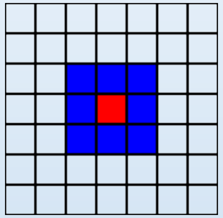
- Four orthogonally adjacent cells.

(ii) Moore neighborhood:

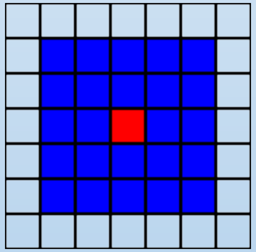
- A central cell and the eight cells which surround it (horizontally, vertically, or diagonally adjacent).
- Conway's Game of Life is a popular version of this model.



von Neumann Neighborhood



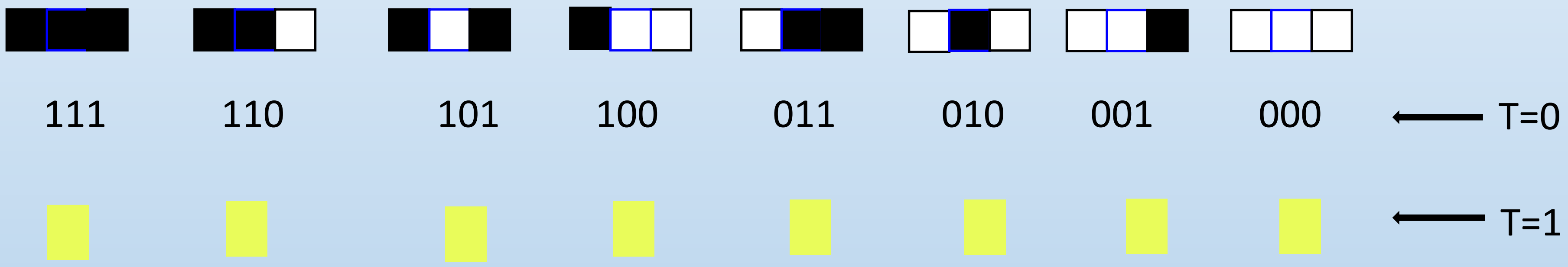
Moore Neighborhood



Extended Moore Neighborhood

One-dimensional Cellular Automata

Every cell and its two neighbors (i.e 3 cells) will be one of the following $2^3 = 8$ possible types.



Every yellow cell above can be filled out with a 0 or a 1 giving a total of $2^8 = 256$ possible update rules.

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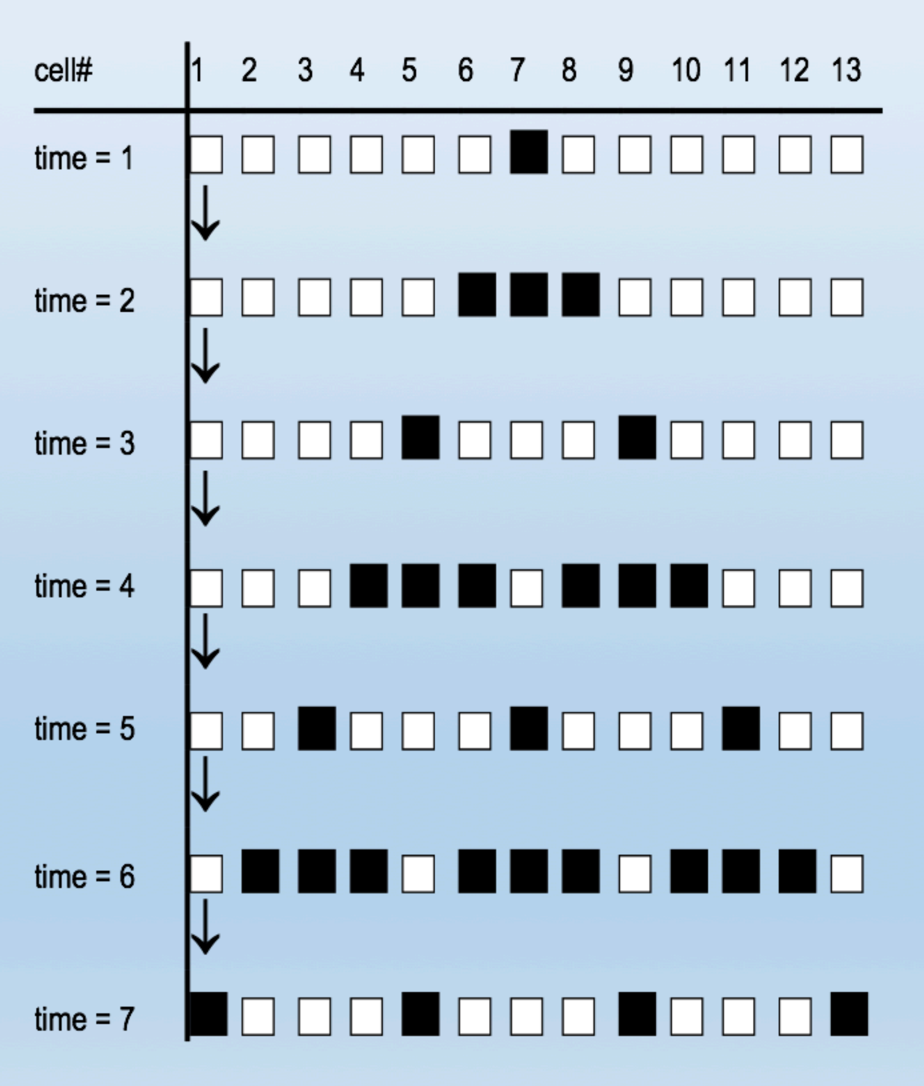
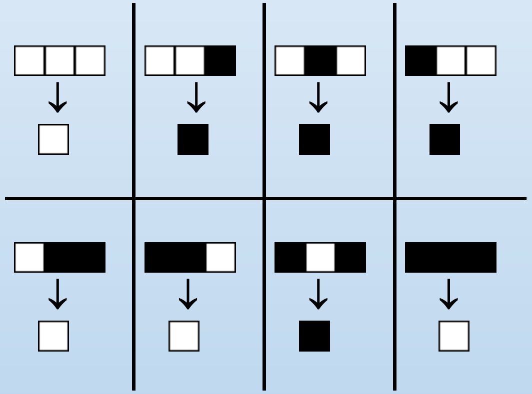
Cellular Automata- Simple 1D example



The rules



Rule 54

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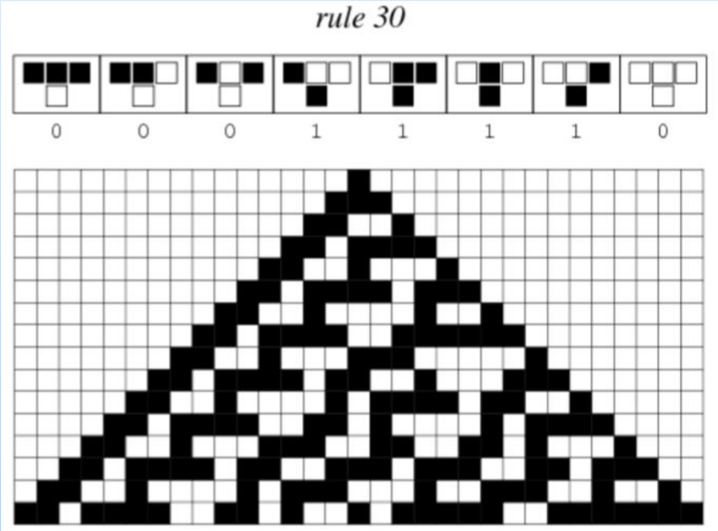
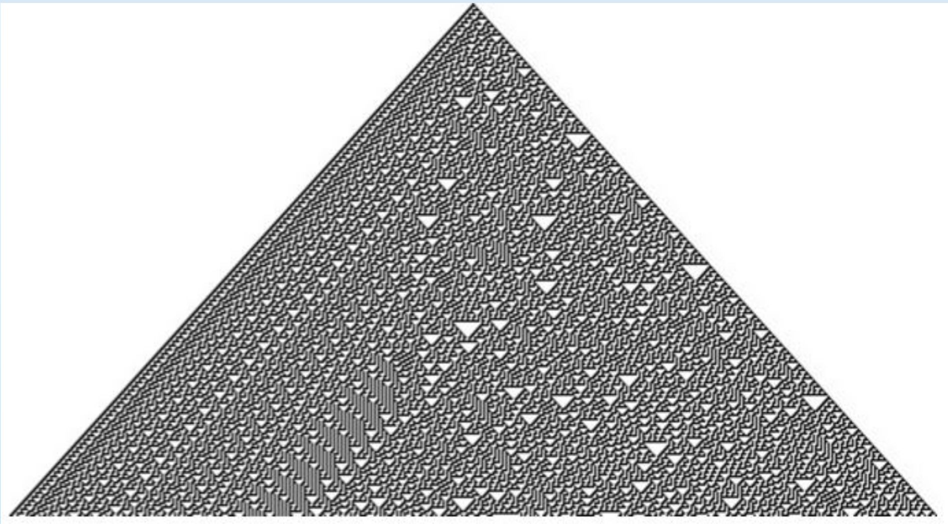
Automata generated using Rule 30 appear in nature, on some shells



After 15 steps (Wolfram 2002, p. 55).



250 iterations of rule 30

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Rule 110

current pattern	111	110	101	100	011	010	001	000
new state for center cell	0	1	1	0	1	1	1	0

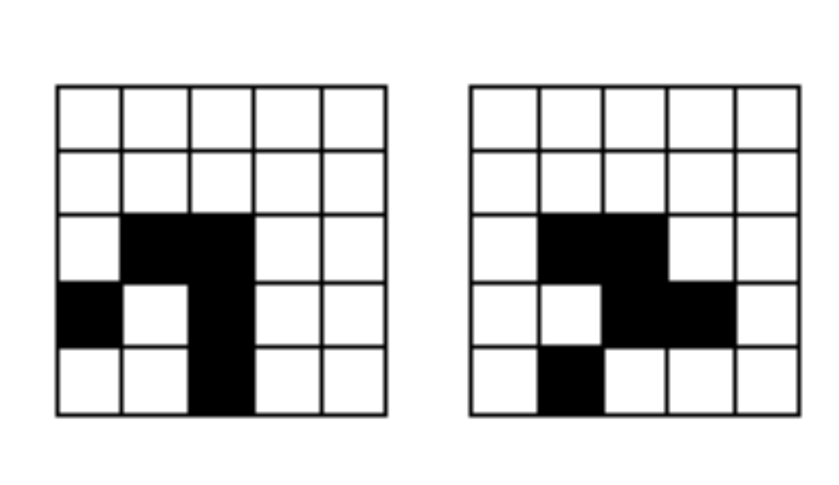
- Rule 110 is known to be Turing complete. This implies that, in principle, any calculation or computer program can be simulated using this automaton.

Conways “Game of Life”

- Conway's Game of Life is a popular version of Moore Neighborhood .

Game of Life provides some rules (live cells by black, and dead cells by white)

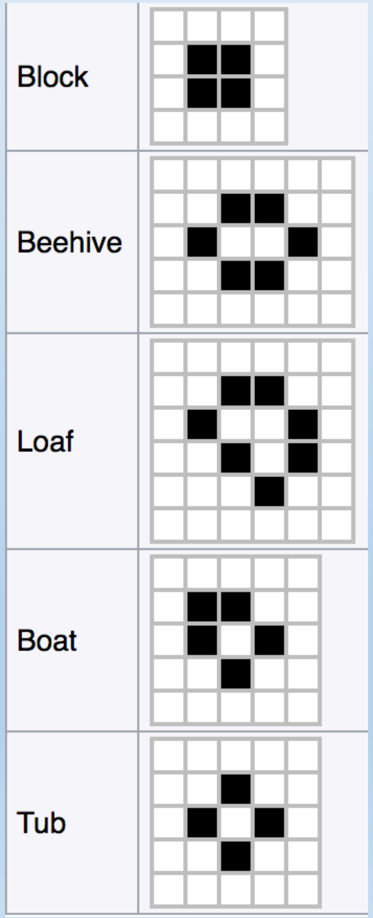
1. Any live cell with fewer than two live neighbors dies, as if by loneliness (under-population).
2. Any live cell with more than three live neighbors dies, as if by overcrowding (over-population).
3. Any live cell with two or three live neighbors lives, unchanged, to the next generation.
4. Any dead cell with exactly three live neighbors comes to life, as if by reproduction.



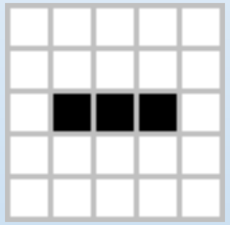
T=1 T=2

Life - Patterns

stable

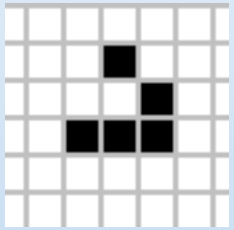


Periodic



Blinker (period 2)

Moving



Glider

Conways “Game of Life”

Also like Rule 110, Conways “Game of Life” is known to be Turing complete.
This implies that, in principle, any calculation or computer program can be simulated using this automaton.

Optical information processing

- Low-loss propagation.
- Low heating.
- High operation speed.
- material nonlinearities.

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Microcavities (Exciton-polariton modes). I. Carusotto and C. Ciuti, “Quantum fluids of light,” Rev. Mod. Phys. 85, 299–374)

↓

Allowed various implementations of individual logic gates and transistors

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Exciton-polariton

- Bistability

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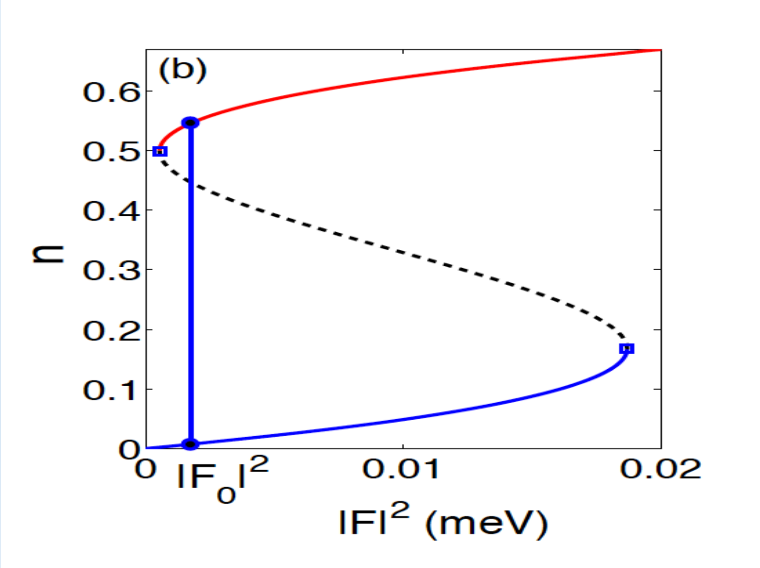
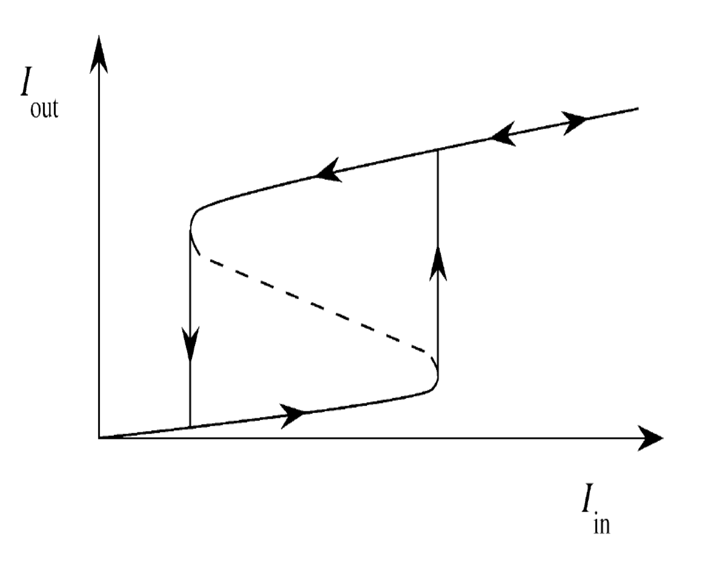
R. Cerna, Y. Leger, T. Para ´ ¨ıso, M. Wouters, et al., Nat. Commun. 4, 2008 (2013).





Aim

- To propose theoretically a photonic Turing machine based on 2-D cellular automata in arrays of nonlinear cavities.
- Rules of Conways "Game of Life" can be realized with a coupled set of 2-D cavities.
- Conways "Game of Life" is known to be Turing complete
- Universal schemes of (classical) computation based on exciton-polaritons in microcavities
- Binary logic states → polariton bistability in microcavities

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Cellular Automata in Photonic Cavity Arrays

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Rule 110

Input cell states:	111	110	101	100	011	010	001	000
Output cell state:	0	1	1	0	1	1	1	0

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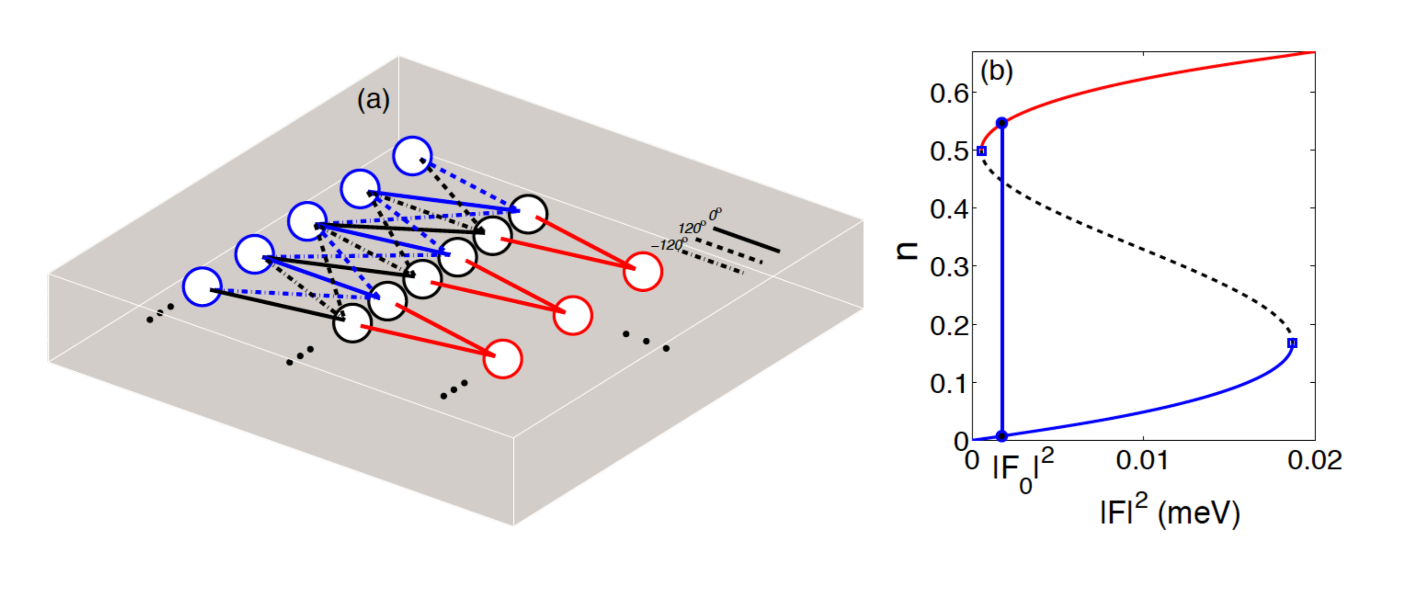
$$i\hbar\frac{d\psi_i}{dt} = \left(-\Delta + \alpha|\psi_i|^2 - i\Gamma\right)\psi_i + F\frac{1}{1+e^{-\frac{t-t_0}{\tau_0}}} + \sum_j J_{ij}\psi_j e^{-i\theta_{ij}}$$

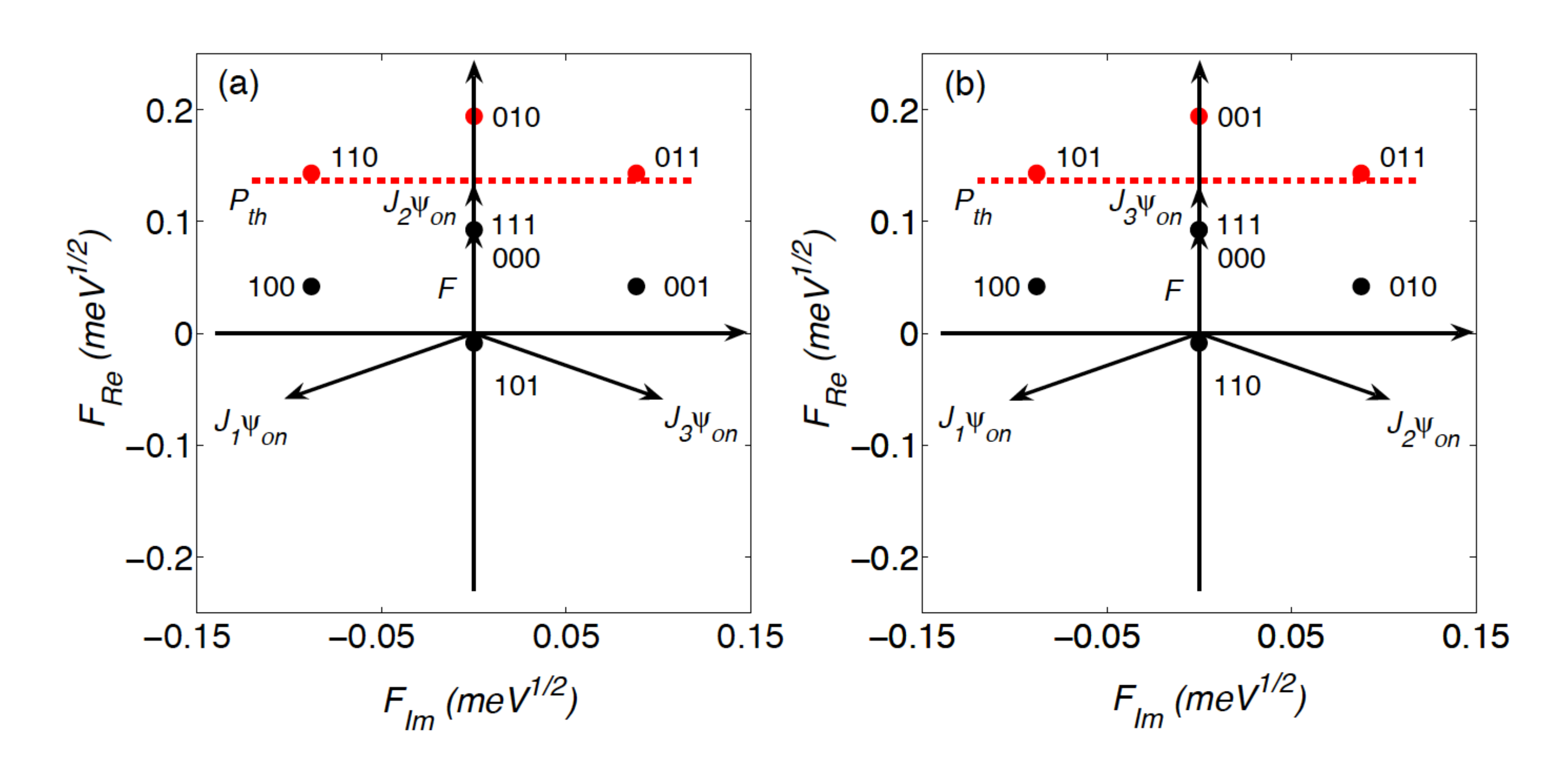
$$|F|^2 = \left[(\alpha n_s - \Delta)^2 + \Gamma^2\right] n_s$$



- Δ : the energy detuning between the laser energy and cavity mode energy
- α is the strength of local (intracavity) Kerr nonlinearity
- Γ is the decay rate
- F the amplitudes of laser excitation of each cavity
- J_{ij} represents a fixed coupling between cavities.

Li, Jing, and T. C. H. Liew. "Cellular automata in photonic cavity arrays." *Optics express* 24.22 (2016): 24930-24937.

$$F+J_1\psi_1 e^{-i\theta_1}+J_2\psi_2 e^{-i\theta_2}+J_3\psi_3 e^{-i\theta_3}$$



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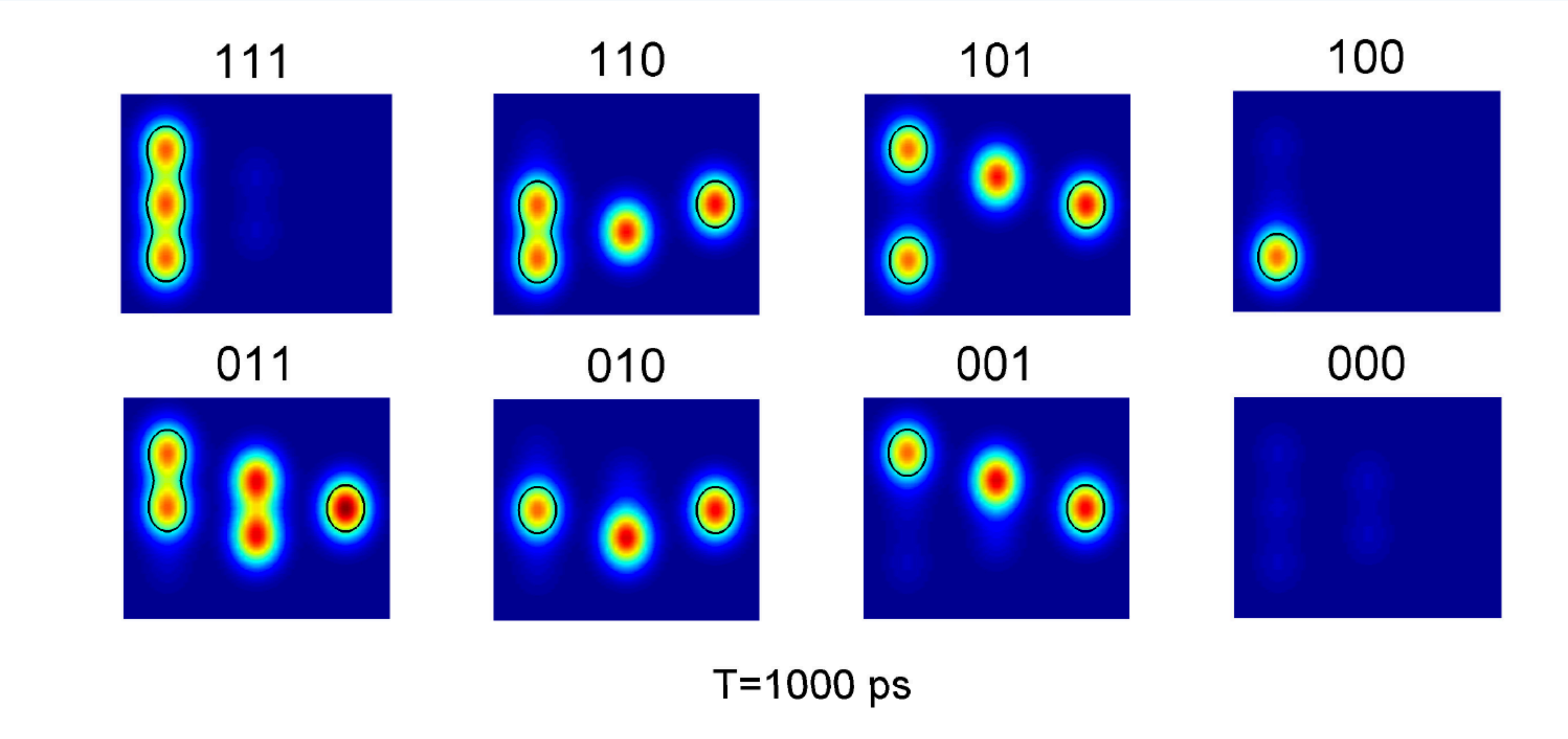

Li, Jing, and T. C. H. Liew. "Cellular automata in photonic cavity arrays." *Optics express* 24.22 (2016): 24930-24937.

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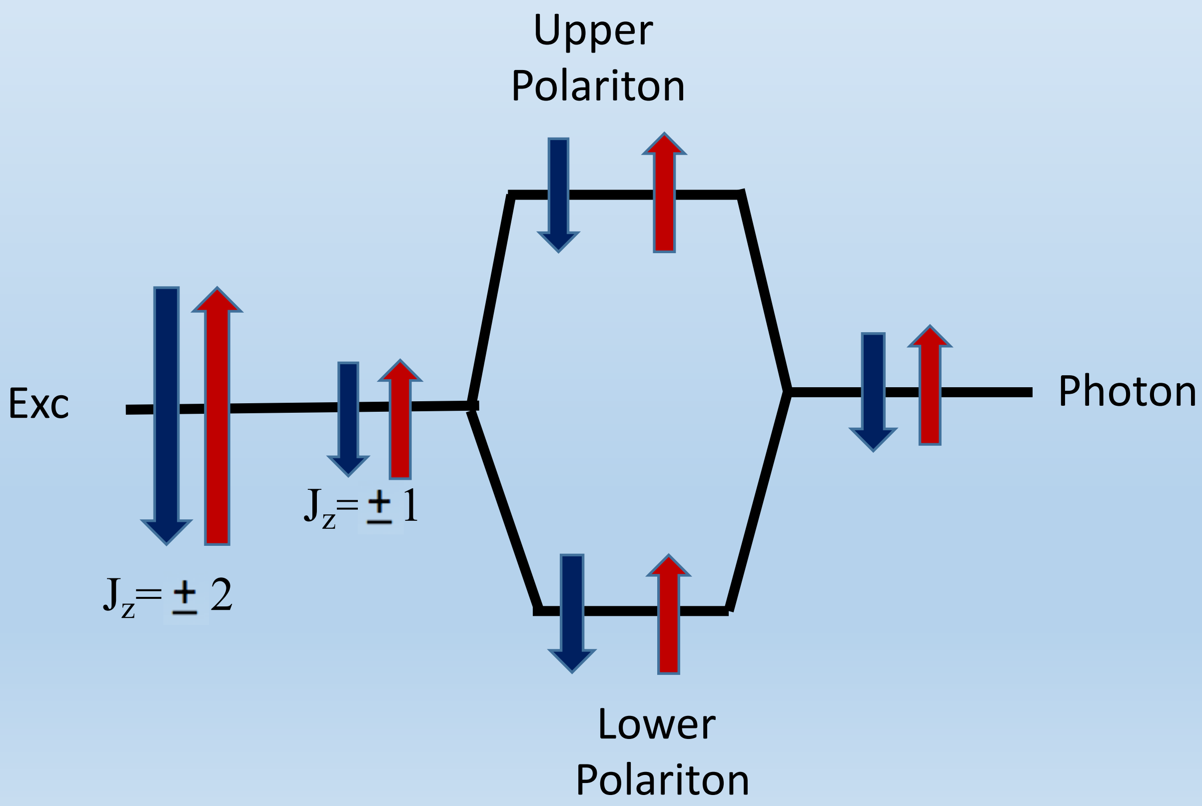
Polariton-spin

Spin: electron $\pm 1/2$

heavy hole $\pm 3/2$

Exciton $J_z = \pm 1$ e ↑↓ h e ↓↑ h

$J_z = \pm 2$ e ↑↑ h e ↓↓ h



- Photon have angular momentum ± 1
- Only $J = \pm 1$ excitons are coupled to light.
- Polaritons have two spin projections:

$J_z = +1$ $\sigma +$

$J_z = -1$ $\sigma -$

} pseudospin

Conclusion

- Conways “Game of Life”
- Exciton-polariton
- Cellular Automata in arrays of 2D Cavities

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Thank you
For Your Attention

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