

Effect of phonon in the condensation of exciton-polaritons

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Quantum light-matter interactions

- ▶ Weak and medium coupling

$$\gamma = \frac{2\pi}{\hbar} |\langle e | H_I(\mathbf{r}) | i \rangle|^2 \underline{\rho(\mathbf{r}, \omega)} \text{ LDOS} \propto \text{Im}[\mathbf{G}_{\hat{p}\hat{p}}(\mathbf{r}, \mathbf{r}, \omega)]$$

- ▶ Strong coupling (dressed quasiparticle)

New excitation, e.g. cavity-atom polaritons and exciton-polaritons

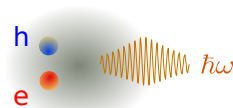
Potential Application

- ▶ Quantum information technology
- ▶ Security, biology and chemistry
- ▶ Imaging with ultra-high resolution
- ▶ Renewable energy, such as solar cell
- ▶ Nanotechnology and Fundamental physics, etc.

Exciton-Polaritons

Strong coupling between exciton and cavity photon

$$a|\text{Exciton}\rangle + b|\text{Photon}\rangle$$



Properties:

- ▶ bosonic statistics below Mott density
- ▶ small effective mass: $\mu \sim 10^{-5}m_e - 10^{-4}m_e$
- ▶ finite life time: $\tau \sim 10^0\text{ps} - 10^2\text{ps}$
- ▶ weak interaction: nonlinearity ($a \neq 0$)

Outline

Motivation

Energy relaxation

Acoustic Phonon — it could be important

Theoretical model

DDGP equation

Quasinormal mode theory

Rate-equation

Numerical calculation and result

Conclusion

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Incoherent pump — how does condensation happen

History: free carries $\xrightarrow{\text{fast}}$ exciton \longrightarrow **exciton-polariton**

- ▶ **Polariton-Polariton Interaction** (instability)

Refs.[A. Kavokin *et al.*, Cavity Polaritons (2003);Phys. Rev. B **82**, 245315 (2010);Phys. Rev. B **91**, 085413 (2015)]

- ▶ Polariton-Phonon Interaction (Optical > Acoustic)
- ▶ Polariton-free carrier interaction (negligible)
- ▶ Polariton-disorder interaction

Refs. [Phys. Rev. Lett. 98, 206402 (2007); J. Phys.: Cond. Mat. **19**, 295208 (2007)]

Could Polariton-Acoustic phonon interaction be important ?

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Incoherently pumped with external electric field

Ref.[M. Klaas, *et al.*, Appl. Phys. Lett., 110, 151103 (2017)]

Optical probing of the Coulomb interactions of an electrically pumped polariton condensate

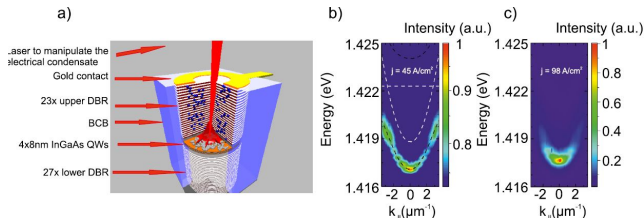
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modulational/fracture
instability is not
found:
**phonon induced
energy relaxation
is important**



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Driven-dissipative Gross-Pitaevskii equation

$$i\hbar\partial_t\Psi = \left[\frac{i\hbar}{2}(Rn - \gamma) - \frac{\hbar^2}{2\mu}\nabla^2 + \alpha|\Psi|^2 + gn\right]\Psi$$

$$\partial_t n = P - (\Gamma + R|\Psi|^2)n$$

At low density limit ($|\Psi|^2$)

$$i\hbar\partial_t\Psi = \left[\frac{i\hbar}{2}(R\beta N_p - \gamma) - \frac{\hbar^2}{2\mu}\nabla^2 + g\beta N_p\right]\Psi(\mathbf{r}).$$

Non-Hermitian system

$$\hat{H} = \frac{i\hbar}{2}(Rn - \gamma) - \frac{\hbar^2}{2\mu}\nabla^2 + gn$$

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Quasi-normal modes (QNM)

- ▶ Eigenmodes $\hat{H}\Psi_{\alpha}(\mathbf{r}) = E_{\alpha}\Psi_{\alpha}(\mathbf{r})$
 - ▶ Hermitian normal modes
 $\text{Im}[E_{\alpha}] = 0$
 - ▶ **Non-Hermitian quasi-normal modes**
 $\text{Im}[E_{\alpha}] \neq 0$ indicates the gain and loss
*widely used in **astronomy** and **electromagnetic system***
- ▶ Localized modes VS extended modes
 - ▶ Localized: physically relevant modes
 - ▶ Extended: physically irrelevant modes
- ▶ Decoupling between localized and extended modes
Their overlap is extremely small

Electromagnetic field is a different story

Normalization of QNM

- ▶ Non-degenerate modes, $\hat{H} = \hat{H}^T$

$$\int \Psi_{\alpha}(\mathbf{r}) \Psi_{\beta}(\mathbf{r}) d\mathbf{r} = \delta_{\alpha\beta}.$$

- ▶ Degenerate, rotational symmetry (2D)

$$\int \Psi_{nm}(\mathbf{r}) \Psi_{kl}(\mathbf{r}) d\mathbf{r} = \delta_{nk} \delta_{ml}$$

with

$$\Psi_{mn}(\mathbf{r}) = \rho_{mn}(r) \exp(\pm im\theta) \exp(-iE_{mn}t/\hbar).$$

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Exciton-phonon interaction

► Hamiltonian

$$H_{\text{phon}} = \sqrt{\frac{\hbar|\mathbf{q}|}{2\rho_D S L u}} (D_e e^{i\mathbf{q}\cdot\mathbf{r}_e} - D_h e^{i\mathbf{q}\cdot\mathbf{r}_h}).$$

► Scattering rate

$$W_{\alpha\rightarrow\beta} = \frac{L_{\text{eff}}}{\hbar^2 u} (n_{\text{phon}}(|\Delta E_{\alpha\beta}^r|) + \Theta(E_{\alpha\beta}^r)) \sum_{\mathbf{q}_{x,y}} |M_{\alpha,pj}(\mathbf{q}_x, \mathbf{q}_y, |\Delta E_{\alpha\beta}^r|)|^2 \\ \times E_{\text{phon}} / \sqrt{E_{\text{phon}}^2 - (\mathbf{q}_x^2 + \mathbf{q}_y^2)(\hbar u)^2}.$$

► Rate equation

$$\partial_t n_\alpha = \frac{2E_\alpha^i}{\hbar} n_\alpha + \sum W_{\beta\rightarrow\alpha} (n_\alpha + 1) n_\beta \\ - \sum W_{\alpha\rightarrow\beta} (n_\beta + 1) n_\alpha - S n_\alpha^2$$

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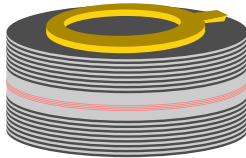
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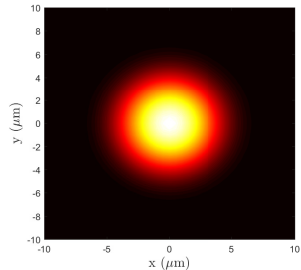
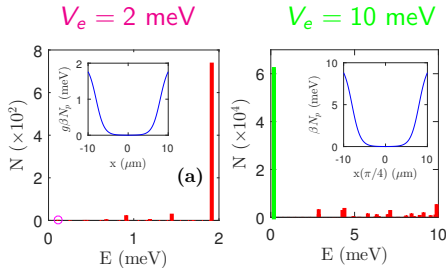
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Electrical pumping with rotational symmetry

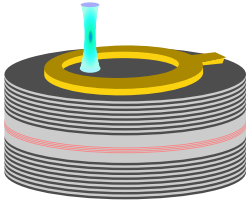


$$V_e^P = g\beta N_p = \frac{V_e}{1 + \exp\left(-(\sqrt{x^2 + y^2} - r_0)/L\right)}$$

$\text{Im}[E_\alpha]$ is “minimum” for ground state

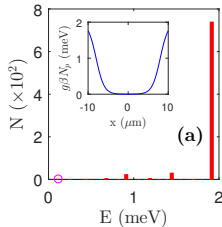


With additional optical pumping



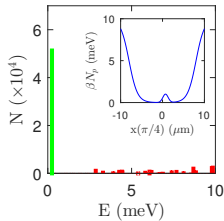
$$V_e = 2 \text{ meV}$$

$$V_o = 0.5 \text{ meV}$$



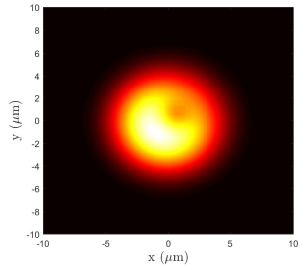
$$V_e = 10 \text{ meV}$$

$$V_o = 1 \text{ meV}$$



$$V = V_e^p + V_o^p \text{ with}$$

$$V_o^p = V_o \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2}{2\Delta}\right)$$



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- ▶ Phonon effect could be important for non-equilibrium condensation
- ▶ We developed a QNM theory to take the effect of the phonon into account intuitively

Ref. “Phonon induced reconfiguration of exciton-polariton condensates in ring traps”, R.C. Ge, C. Schneider, and T. C. H. Liew, in review.

acknowledgement

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