Zero-energy vortices in honeycomb lattices

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Physics of Exciton-Polaritons in Artificial Lattices

Daejeon, 16 May, 2017


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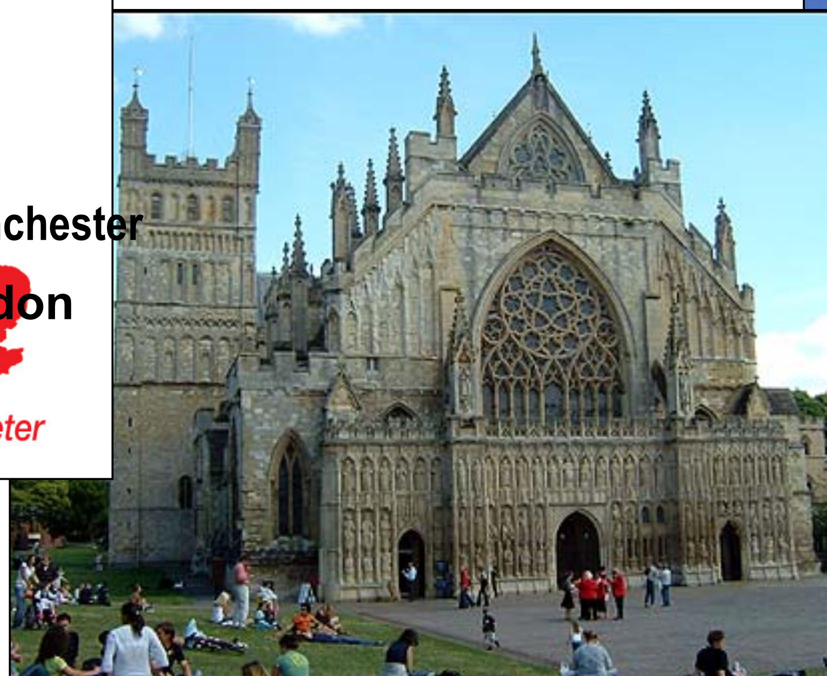
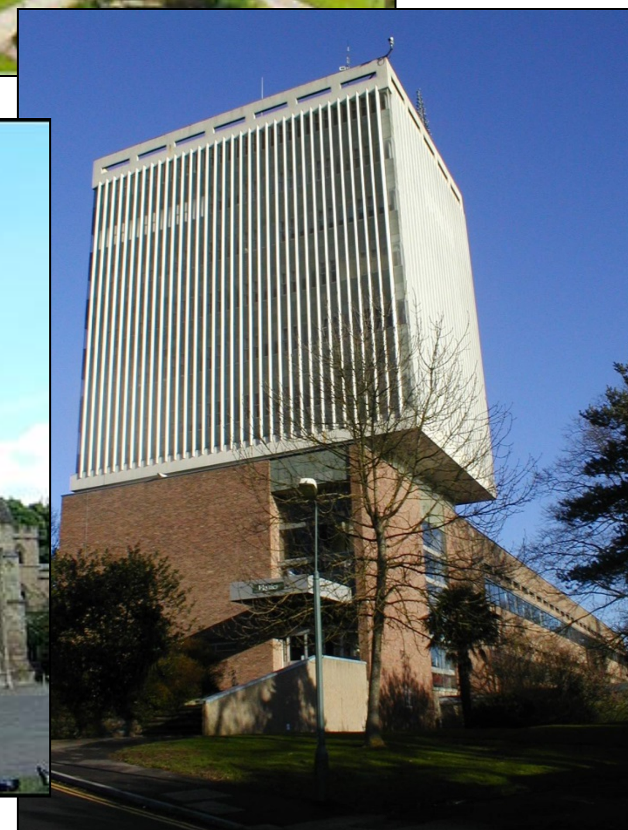




Dirac equation in high energy physics

Relativistic theory of an electron - Dirac equation

$$[\gamma_\mu(-i\hbar\partial_\mu - eA_\mu) - m]\Psi = 0$$

4x4 matrices

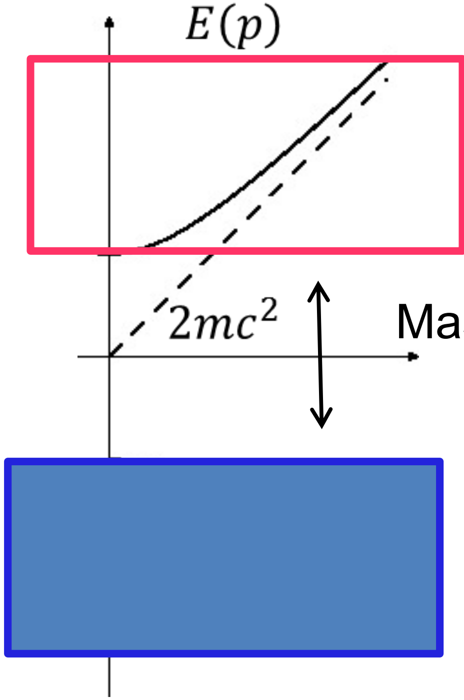
Wavefunction is a 4-component vector

Free solutions:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

A quantum relativistic particle has 4 internal states;
two of these can be identified
with particle's spin.

The other two states describe
an <u>antiparticle</u> (or a hole).



Electron states, empty
(conduction band)

Mass gap, no free states

Positron (or hole) states
forming Dirac sea
(valence band)

Dirac material family in condensed matter

Material	Pseudospin	Energy scale (eV)
Graphene, Silicene, Germanene	Sublattice	1–3 eV
Artificial Graphenes	Sublattice	10^{-8}–0.1 eV
Hexagonal layered heterostructures	Emergent	0.01–0.1 eV
Hofstadter butterfly systems	Energent	0.01 eV
Graphene-hBN heterostructures in high magnetic fields		
Band inversion interfaces	Spin-orbit ang. mom.	0.3 eV
SnTe/PbTe, CdTe/HgTe, PbTe		
2D Topological Insulators	Spin-orbit ang. mom.	< 0.1eV
HgTe/CdTe, InAs/GaSb, Bi bilayer, ...		
3D Topological Insulators	Spin-orbit ang. mom.	$\lesssim 0.3$eV
$Bi_{1-x}Sb_x$, Bi_2Se_3, strained HgTe, Heusler alloys, ...		
Topological crystalline insulators	orbital	$\lesssim 0.3$eV
SnTe, $Pb_{1-x}Sn_xSe$		
d-wave cuprate superconductors	Nambu pseudospin	$\lesssim 0.05$eV
^{3}He	Nambu pseudospin	0.3 μeV
3D Weyl and Dirac semimetals	Energy bands	Unclear
Cd_3As_2, Na_3Bi		

Table 1. Table of Dirac materials indicated by material family, pseudospin realization in the Dirac Hamiltonian, and the energy scale for which the Dirac spectrum is present without any other states.

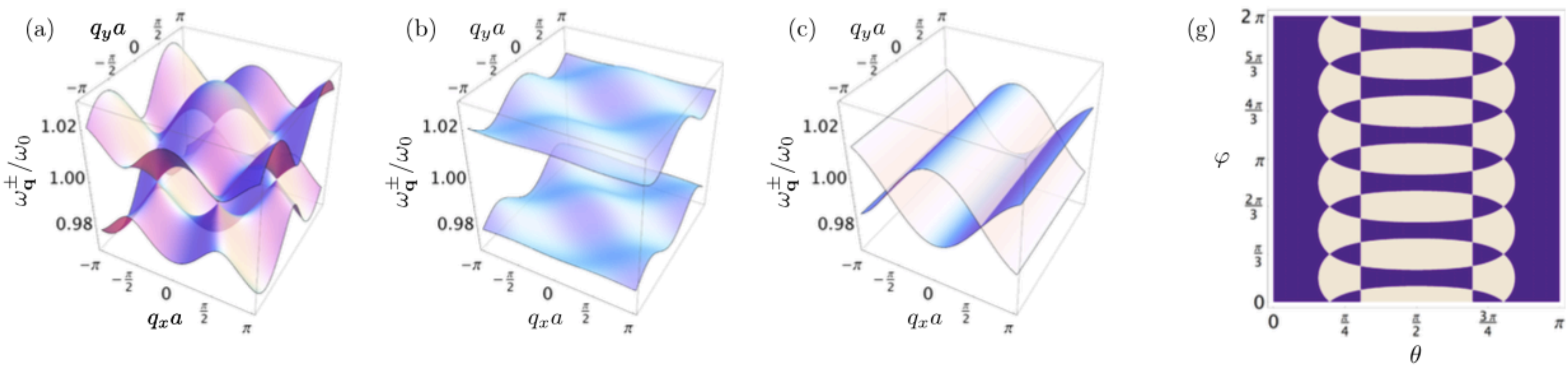
Wehling et al., Advances in Physics **63**, 1 (2014)

Artificial graphene research in Exeter (theory)

PRL **110,** 106801 (2013) PHYSICAL REVIEW LETTERS week ending 8 MARCH 2013

Dirac-like Plasmons in Honeycomb Lattices of Metallic Nanoparticles

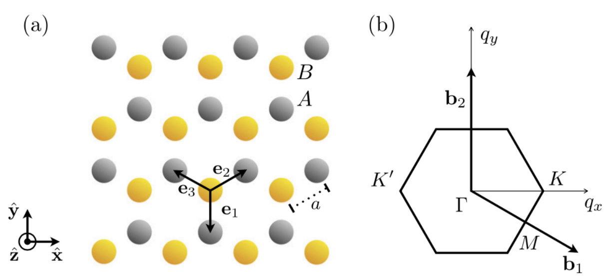
Guillaume Weick,[1] Claire Woollacott,[2] William L. Barnes,[2] Ortwin Hess,[3] and Eros Mariani[2]

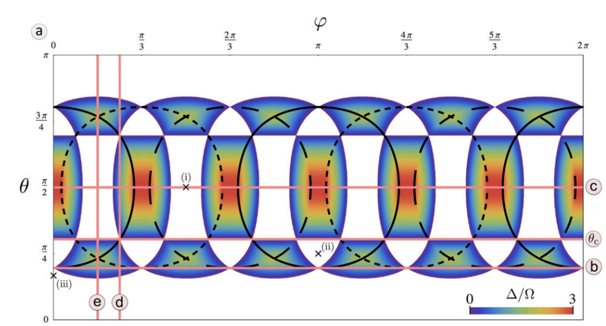


IOP Publishing *2D Mater.* **2** (2015) 014008

2D Materials Dirac plasmons in bipartite lattices of metallic nanoparticles

Thomas Jebb Sturges[1], Claire Woollacott[1], Guillaume Weick[2] and Eros Mariani[1]





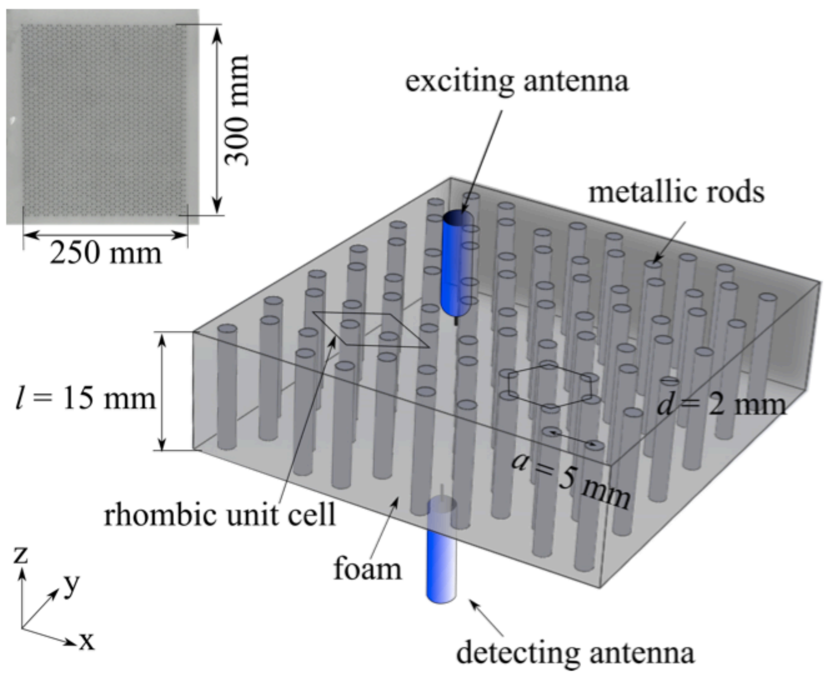
Artificial graphene research in Exeter (experiment)

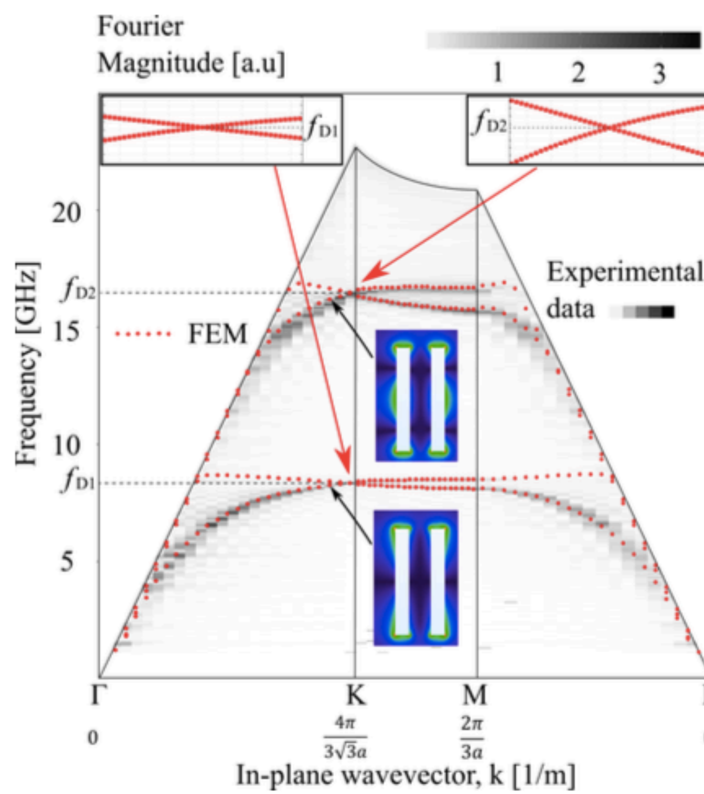
Gapless States in Microwave Artificial Graphene

Yulia N. Dautova,[1, a)] Andrey V. Shytov,[1] Ian R. Hooper,[1] J. Roy Sambles,[1] and Alastair P. Hibbins[1]
Department of Physics and Astronomy, University of Exeter, Exeter, Devon EX4 4QL, United Kingdom.

EPSRC Centre for Doctoral Training in Metamaterials
XM²

A microwave analogue of graphene comprised of cylindrical metallic rods arranged in a honeycomb array is fabricated. Dispersion curves of the bound electromagnetic eigenmodes of the system were experimentally determined by measuring the electric near-fields just above the surface. Two linear crossings are evident in these dispersion curves at each K and K' points of the Brillouin zone, mimicking the well-celebrated Dirac cones in graphene.



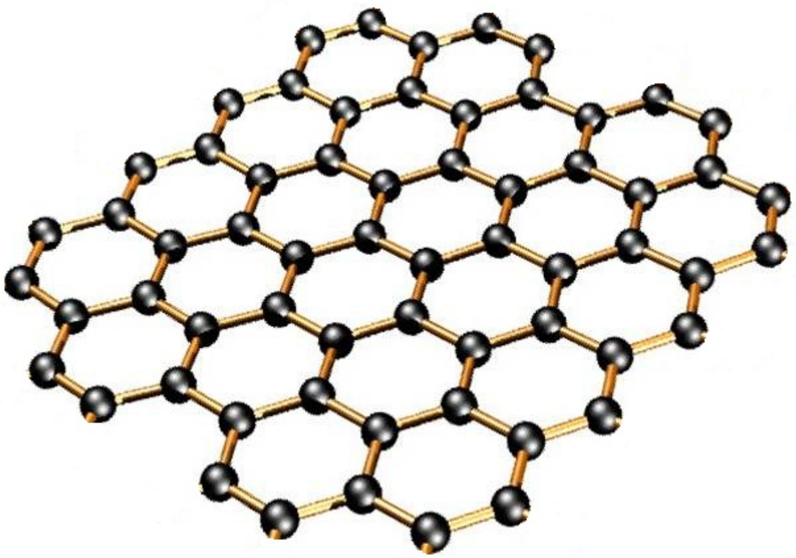


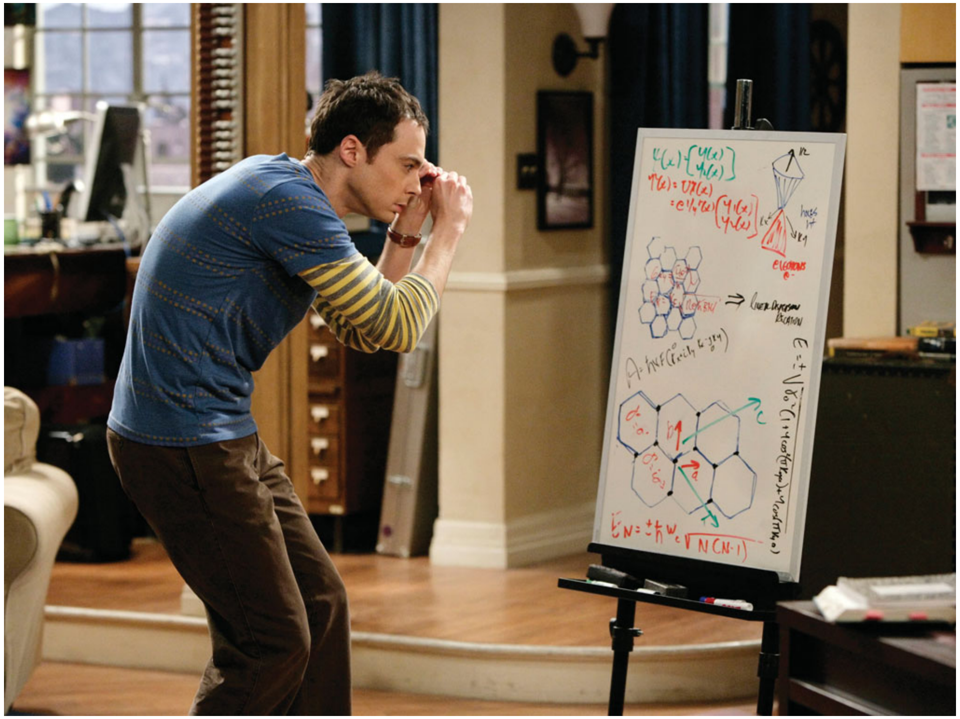
EPSRC
Engineering and Physical Sciences Research Council

UNIVERSITY OF EXETER

The rise of graphene

The 2010 Physics Nobel Prize for Geim & Novoselov – for a review see their Nobel lectures in RMP





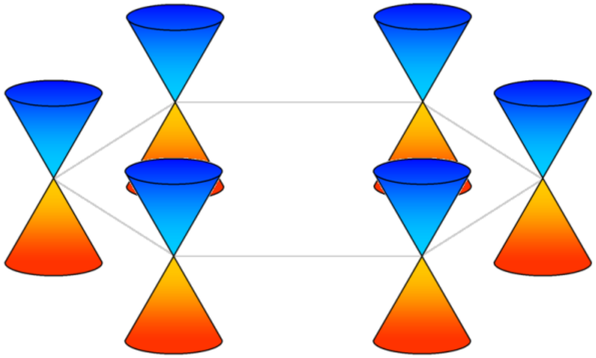
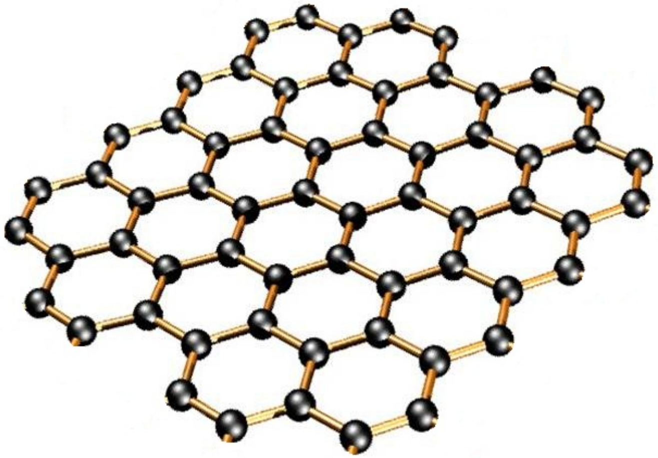
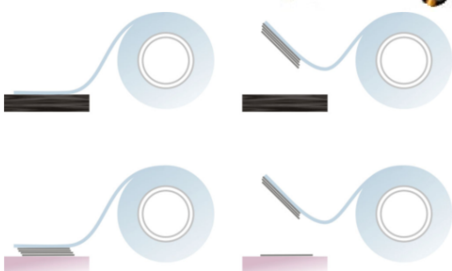
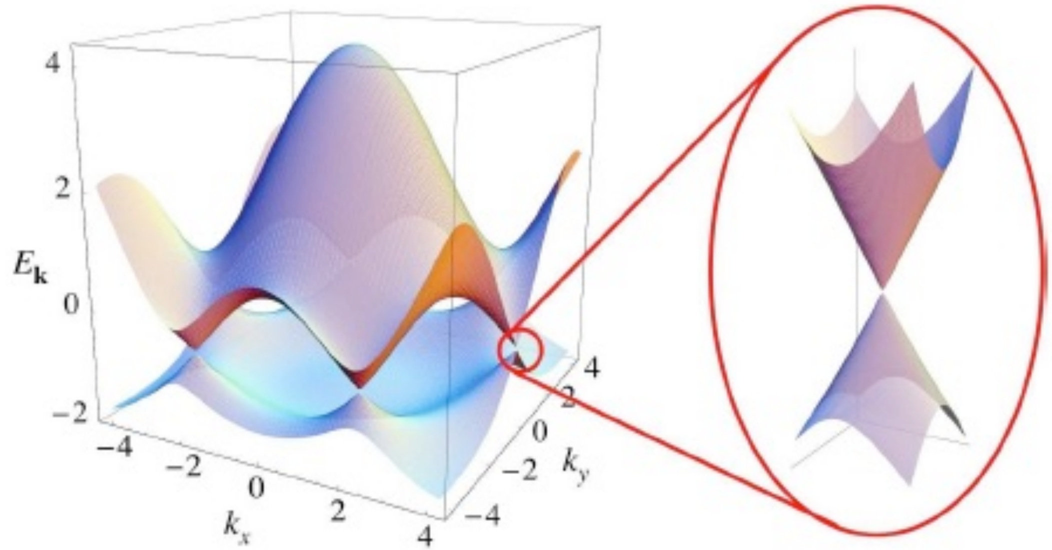


FIG. 11 (color). Dr. Sheldon Cooper (Jim Parsons) "…either isolating the terms of his formula and examining them individually or looking for the alligator that swallowed his hand after Peter Pan cut it off." From *The Big Bang Theory*, series 3, episode 14 "The Einstein Approximation." Photo: Sonja Flemming/CBS ©2010 CBS Broadcasting Inc.





Novoselov et al.
Science **306**, 666
(2004)



P.R. Wallace,
Phys. Rev. **71**, 622
(1947)

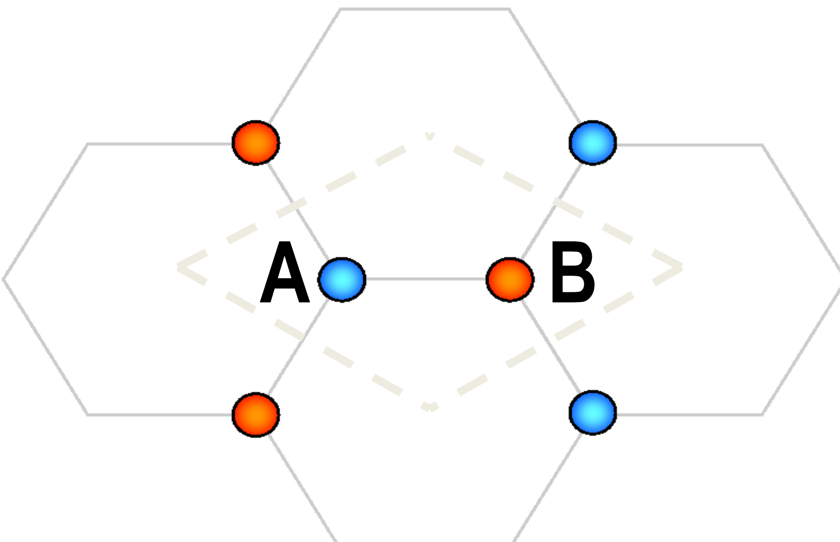
Graphene dispersion

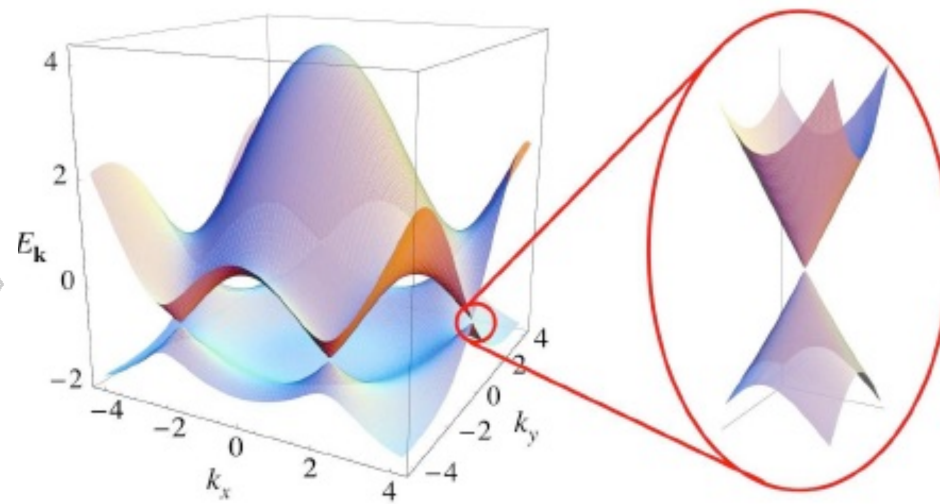
Unconventional QHE; huge mobility (suppression of backscattering), universal optical absorption…
Theory: use of 2D relativistic QM, optical analogies, Klein paradox, valleytronics…

Dispersion Relation

$$E = \pm\gamma_0\sqrt{|f(\underline{k})|}$$

$$f(\underline{k}) = e^{i\left(\frac{a}{\sqrt{3}}k_x\right)} + 2e^{i\left(-\frac{a}{2\sqrt{3}}k_x\right)}\cos\left(k_y\frac{a}{2}\right)$$





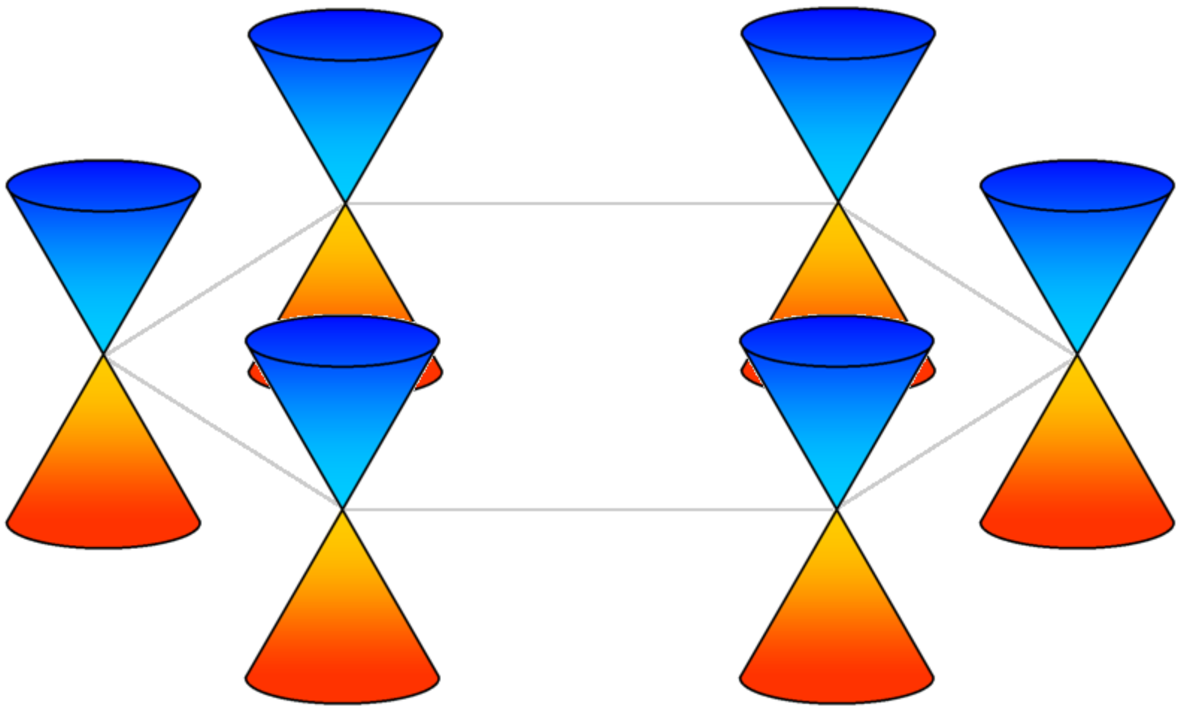
$$E = \pm\hbar v_{\mathrm{F}}|\mathbf{q}| \qquad c/v_{\mathrm{F}} \approx 300$$

P. R. Wallace “The band theory of graphite”, Phys. Rev. **71**, 622 (1947)

70-year old achievement of condensed matter physics

“Dirac Points”

Expanding around the K points in terms of small q

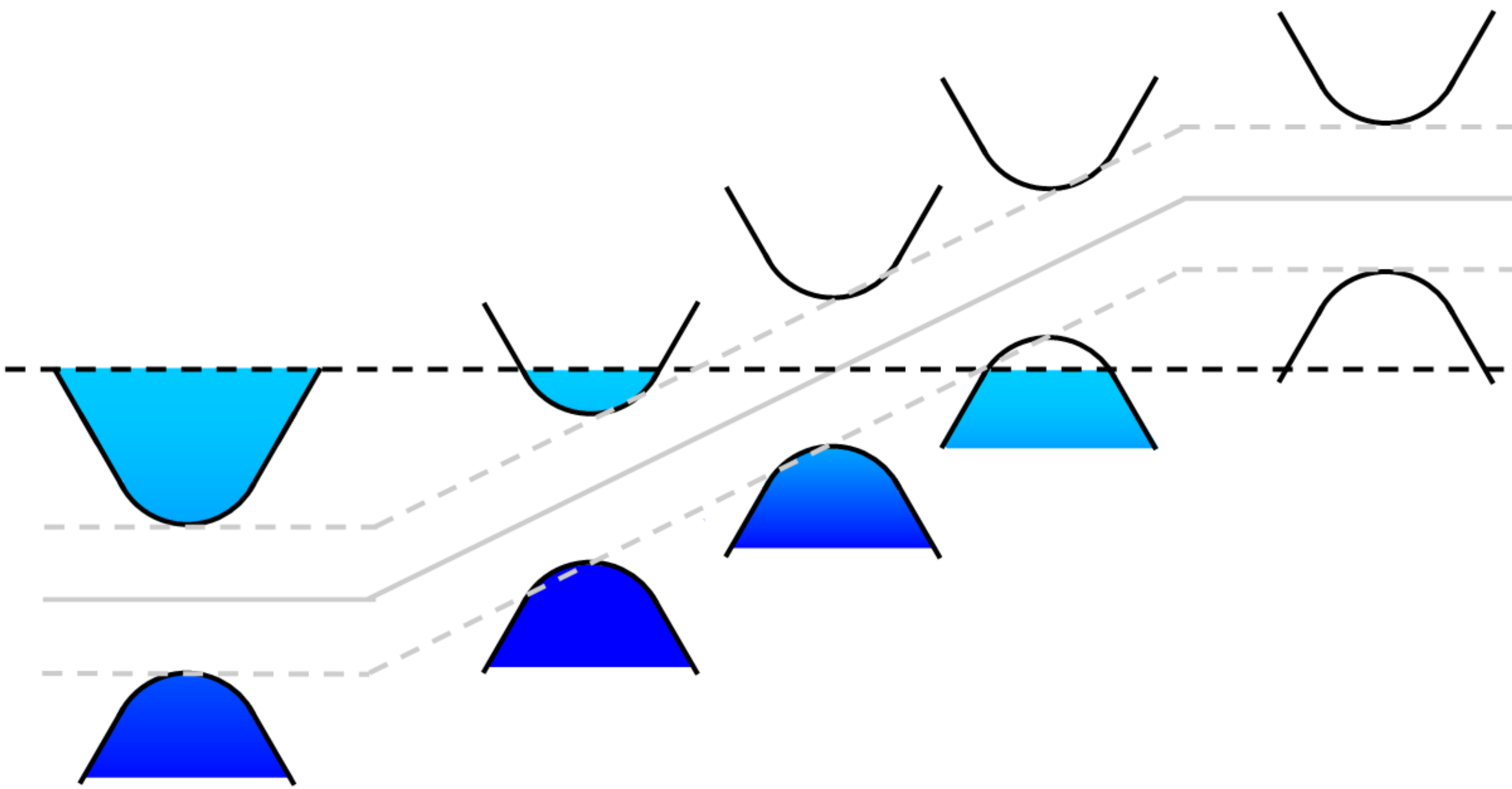


$$\widehat{H} = \hbar v_{\mathrm{F}} \begin{pmatrix} 0 & \widehat{q}_x - i\widehat{q}_y \\ \widehat{q}_x + i\widehat{q}_y & 0 \end{pmatrix} \quad C \equiv \frac{\boldsymbol{q}\widehat{\boldsymbol{\sigma}}}{|\boldsymbol{q}|}, \ \langle C \rangle = \pm 1$$

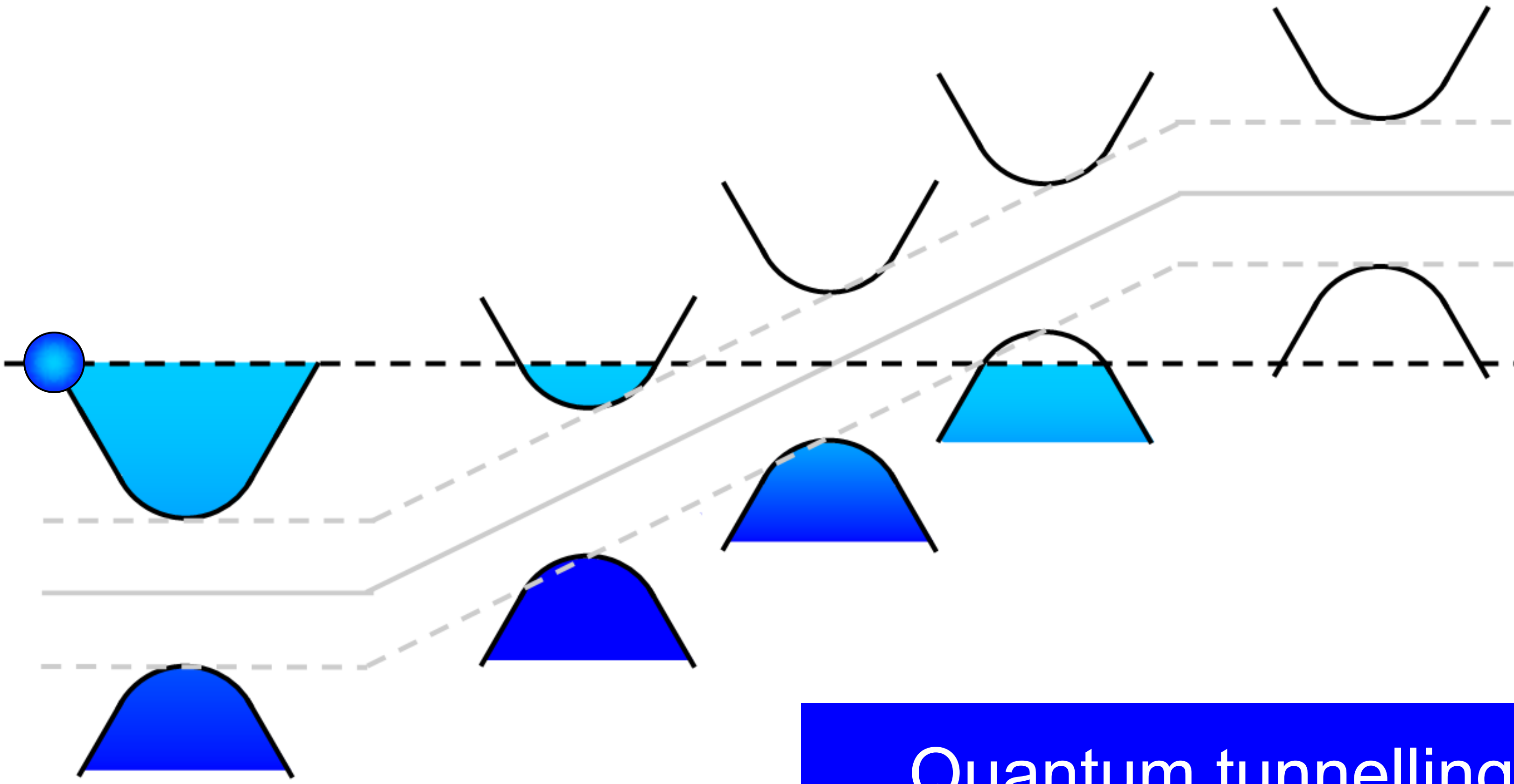
Electrostatic confinement is somewhat difficult…

Quantum tunnelling

in a conventional semiconductor system



Quantum tunnelling
in a conventional semiconductor system

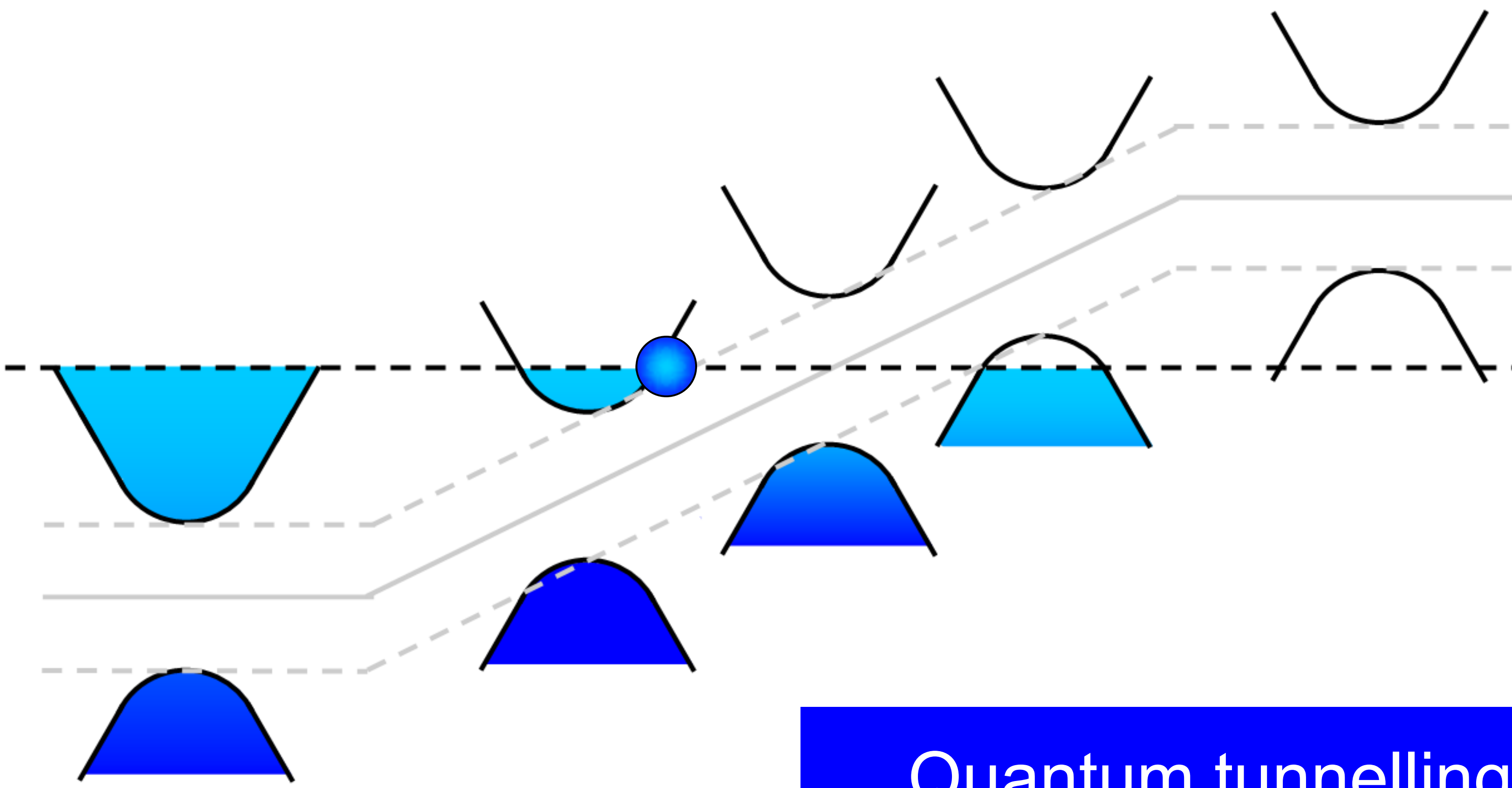


Quantum tunnelling

in a conventional system

Quantum tunnelling

in a conventional semiconductor system

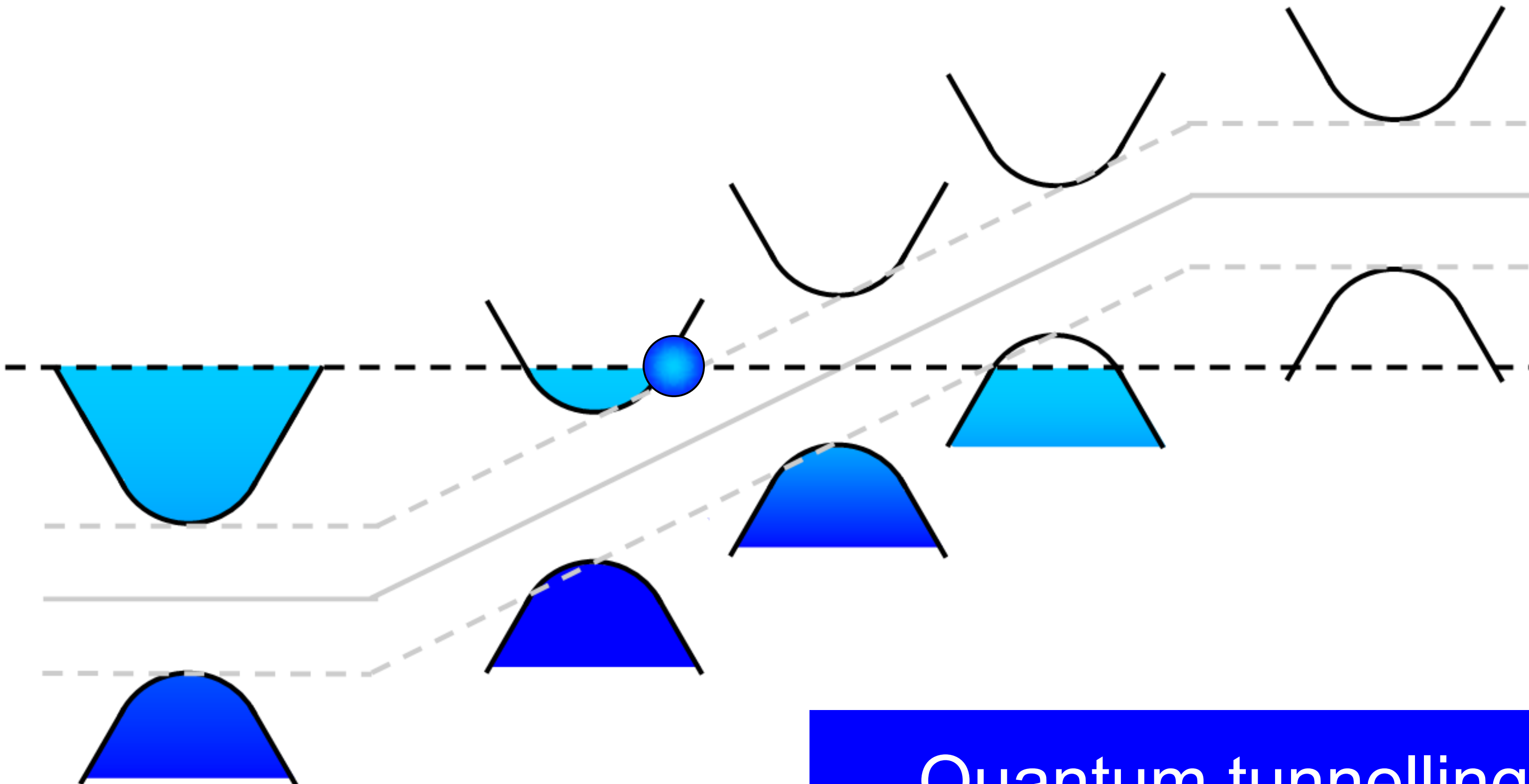


Quantum tunnelling

in a conventional system

Quantum tunnelling

in a conventional system

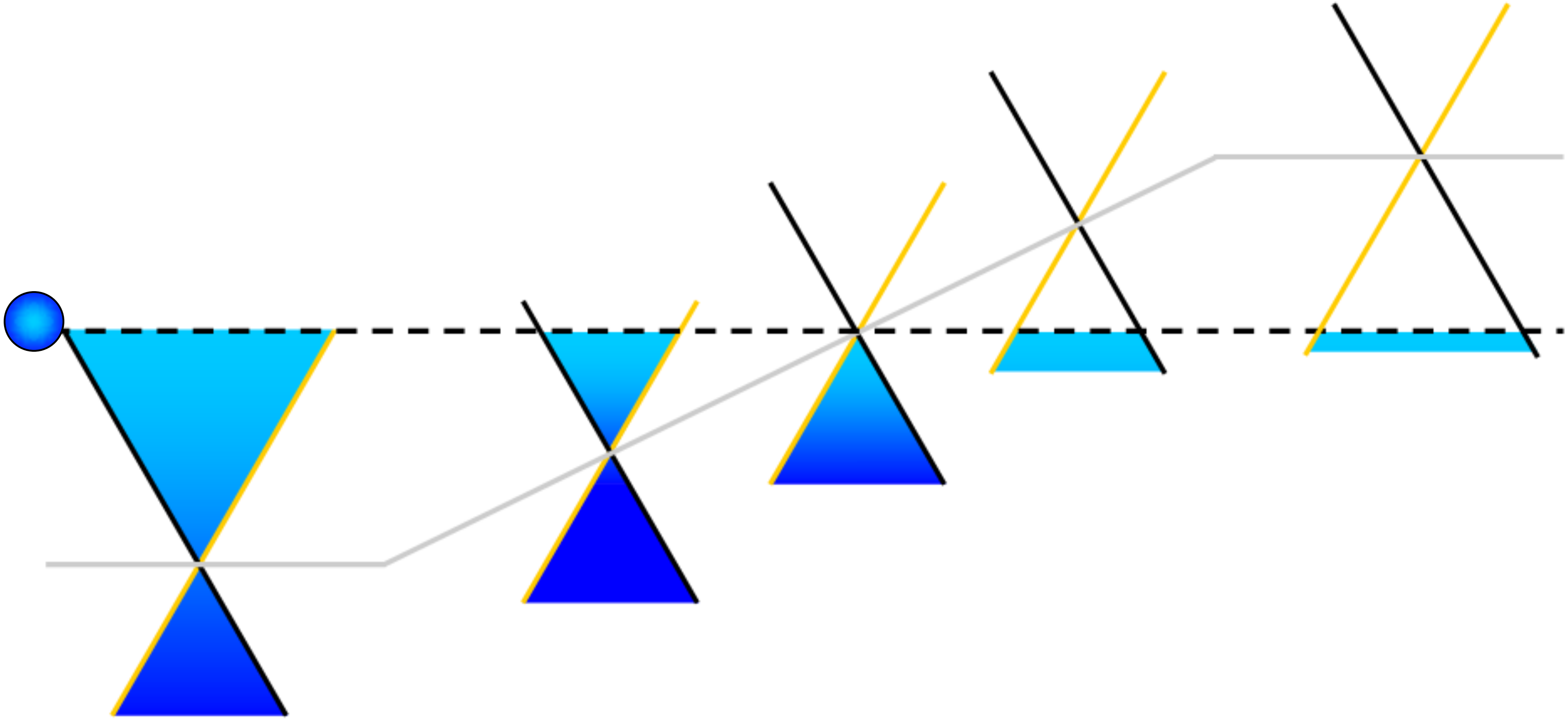


Quantum tunnelling

in a conventional system

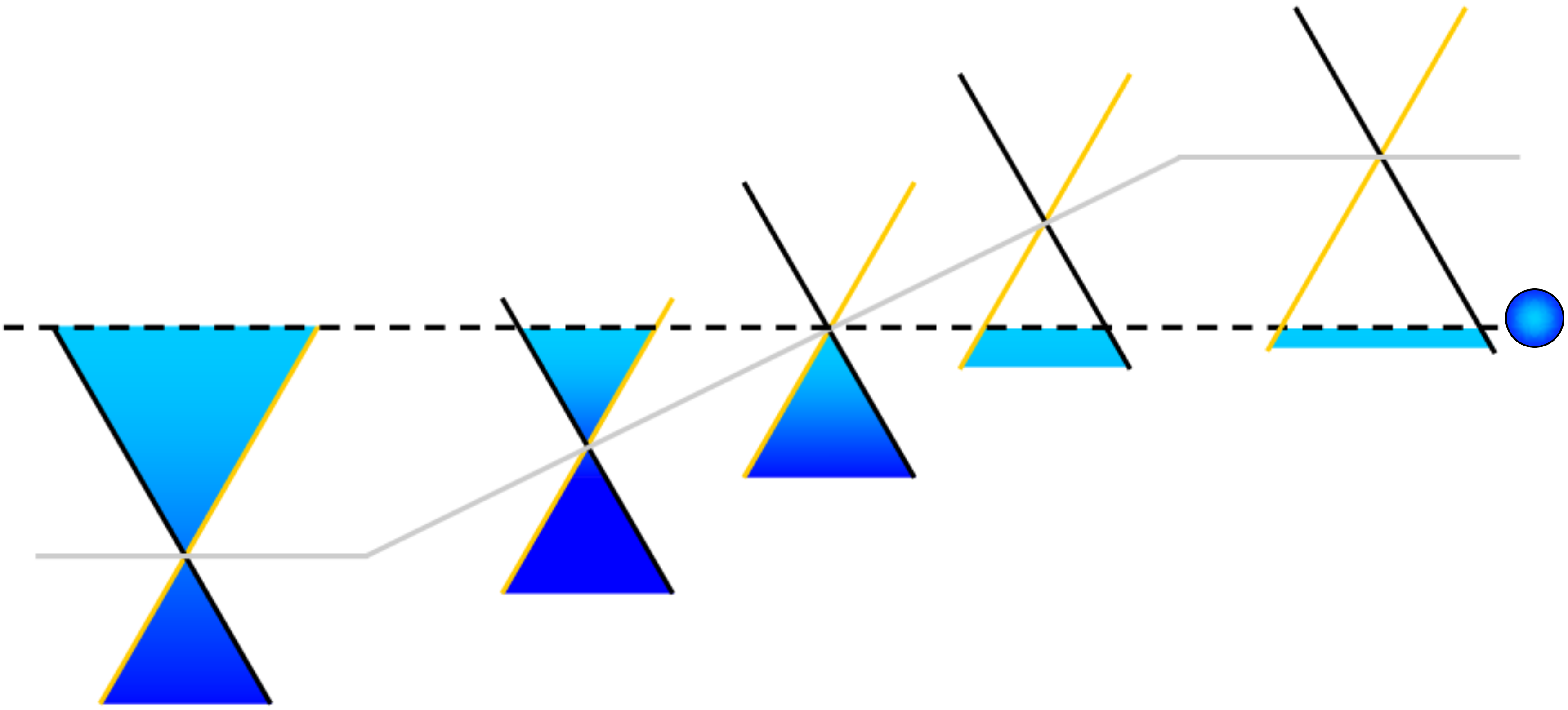
Klein tunnelling

in a graphene p-n junction



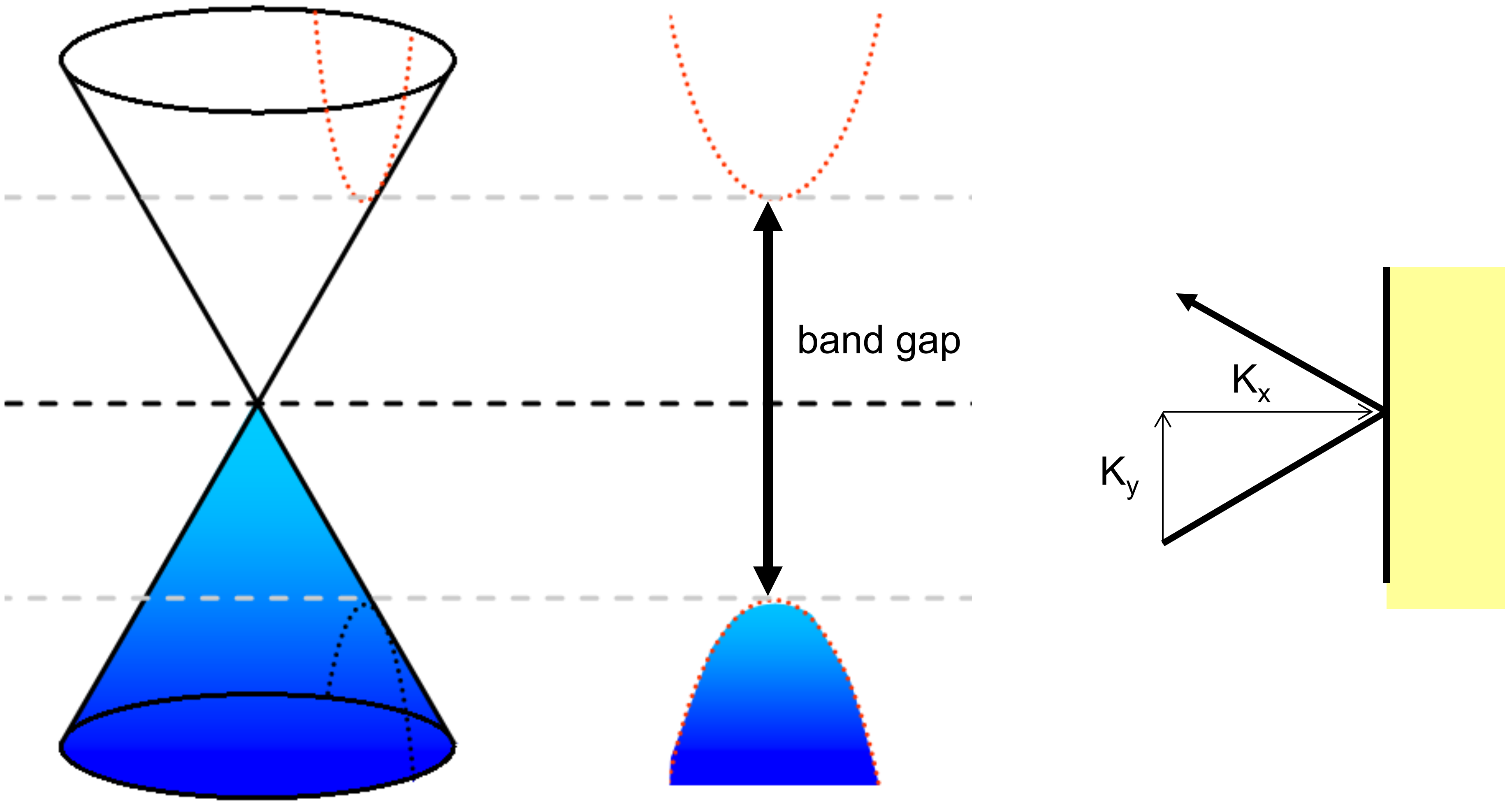
Klein tunnelling

in a graphene p-n junction



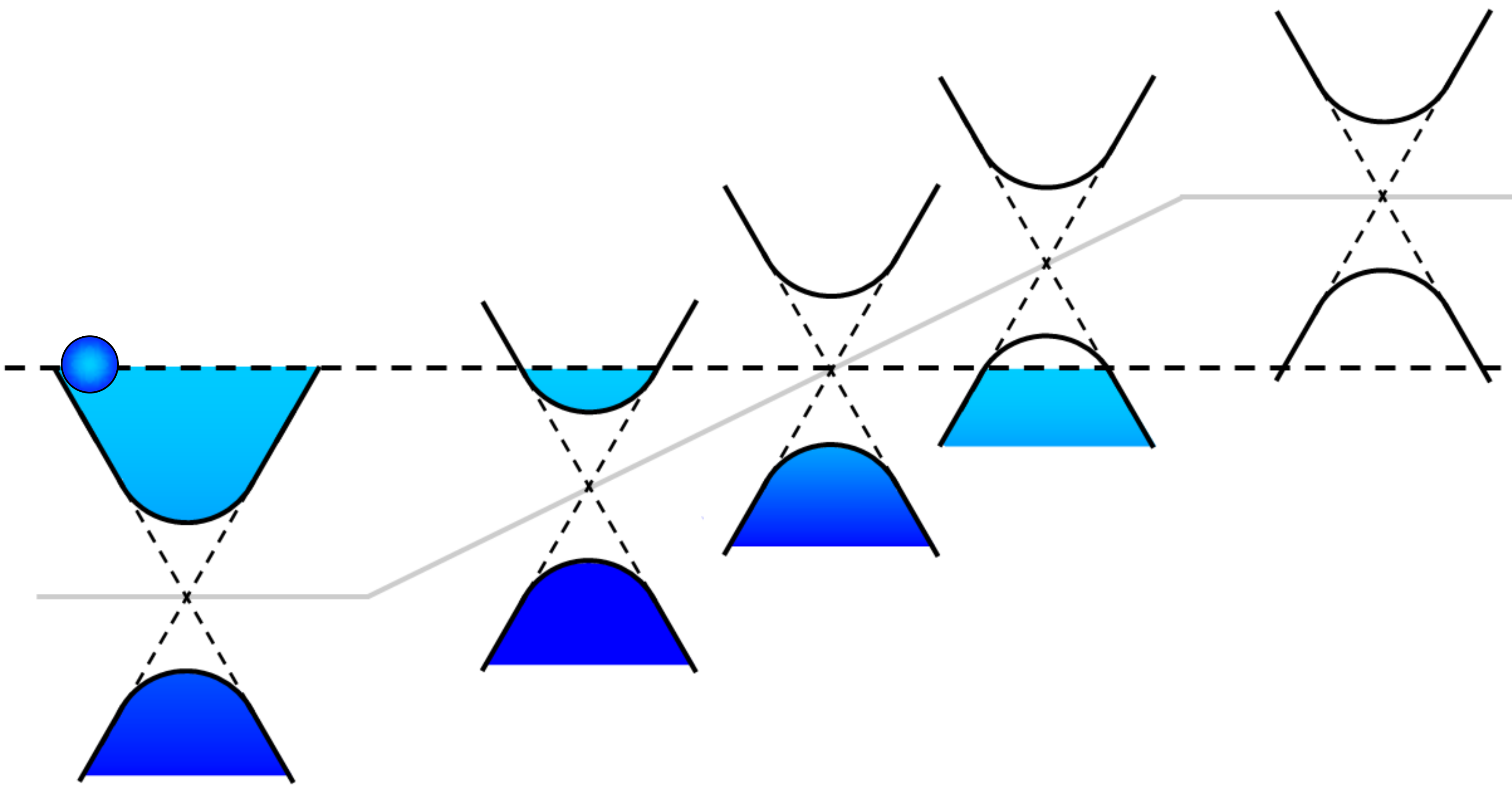
No bound states?
No off switch?

Non-incident electrons

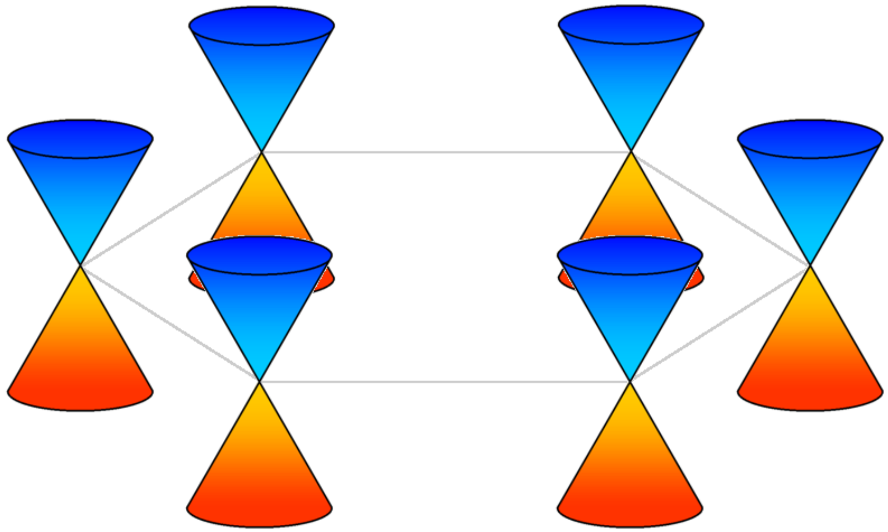


Non-incidence tunneling

in a graphene p-n junction



Mathematically



Free particle:

$$\widehat{H} = \hbar v_{\mathrm{F}} \begin{pmatrix} 0 & \widehat{q}_x - i\widehat{q}_y \\ \widehat{q}_x + i\widehat{q}_y & 0 \end{pmatrix}$$

Normal incidence: $q_y = 0$

Arbitrary 1D barrier: $V(x)$

$$(V(x) - E)\Psi_A - i\hbar v_{\mathrm{F}} \frac{\partial \Psi_B}{\partial x} = 0$$

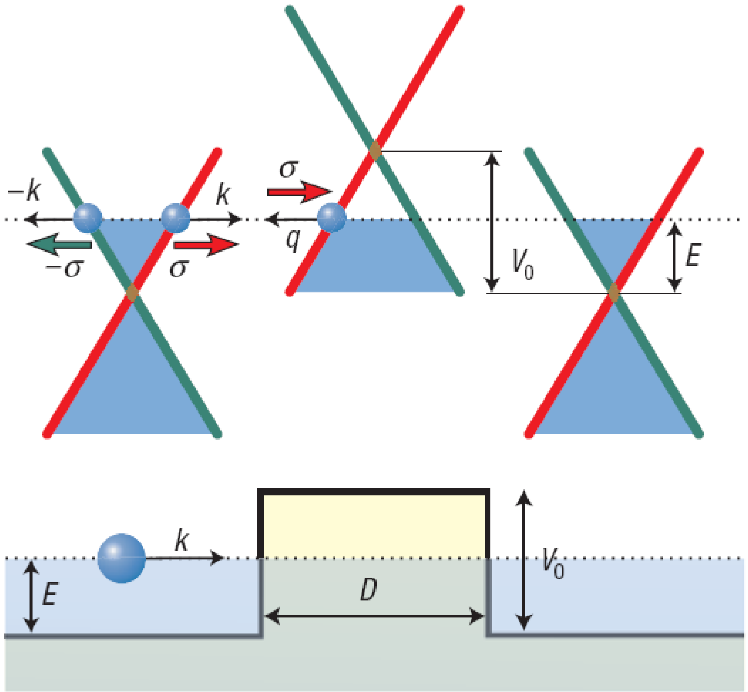
$$(V(x) - E)\Psi_B - i\hbar v_{\mathrm{F}} \frac{\partial \Psi_A}{\partial x} = 0$$

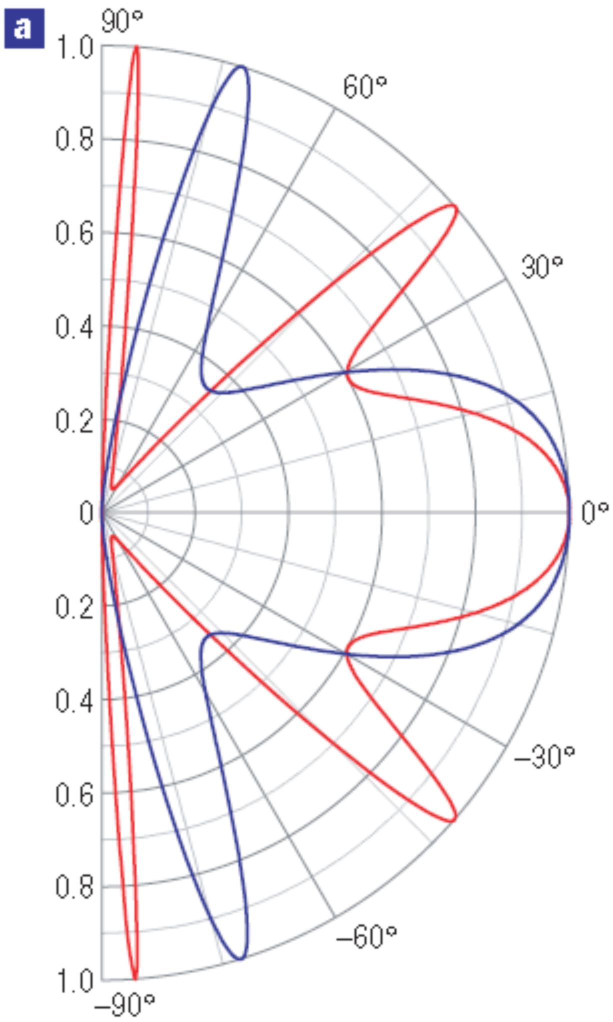
$\pm \Rightarrow$

$$(V(x) - E)\Psi_\pm - i\hbar v_{\mathrm{F}} \frac{\partial \Psi_\pm}{\partial x} = 0$$

$$\Psi_\pm(x \to \pm\infty) \propto e^{+iqx} \exp\left(\frac{-i}{\hbar v_{\mathrm{F}}} \int_a^b V(x) \mathrm{d}x \right)$$

Transmission probability through a square barrier





Transmission probability for a fixed electron and two different hole densities (barrier heights). From: M.I.Katsnelson, K.S.Novoselov, A.K.Geim, Nature Physics 2, 620 (2006).

Fully-confined states?

Circularly-symmetric potential $V(r)$

$$\psi(r,\vartheta) = \begin{pmatrix} \mathrm{e}^{im\vartheta}\chi_A(r) \\ \mathrm{e}^{i(m+1)\vartheta}\chi_B(r) \end{pmatrix}$$

$$\begin{pmatrix} V(r) & -i\frac{\partial}{\partial r} - \frac{i(m+1)}{r} \\ -i\frac{\partial}{\partial r} + \frac{im}{r} & V(r) \end{pmatrix} [\chi] = E\,[\chi]$$

$E \neq 0$ – confinement is not possible for any fast-decaying potential…

“Theorem” - no bound states

$$\hat{H} = v_{\mathrm{F}} \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} + V(r)$$

$$\left\{-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{n^2}{r^2} - (E + v_0)^2\right\}\chi_1 = 0, \quad r < a,$$

$$\left\{-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{n^2}{r^2} - E^2\right\}\chi_1 = 0, \quad r \geq a.$$

Inside well

$$J_n(|E + v_0|r)$$

Outside well

$$J_n(|E|r) \quad \text{and} \quad N_n(|E|r)$$

Equation (20) involves only the square of energy. Therefore, its solutions do not depend on the sign of E. These solutions are equivalent to the scattering states for the ordinary radial Schrödinger equation with $E > 0$. Hence, it follows that, in such a potential well, bound states are absent. We stress that this conclusion does not depend on the depth and width of the well; i.e., a two-dimensional localization of quasiparticles in graphene (quantum dot) is fundamentally impossible (evidently,

With asymptotics

$$J_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\cos\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\mathrm{Im}(z)|} O(|z|^{-1})\right)$$

$$Y_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\sin\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\mathrm{Im}(z)|} O(|z|^{-1})\right)$$

Tudorovskiy and Chaplik, JETP Lett. **84**, 619 (2006)

$\boxed{E = 0}$ **long-range behaviour:**

$$V(r) = V_0 r^{-p}, \; p > 1.$$

Always true for gated structures

$$\chi_A'' + \left(\frac{1+p}{r}\right)\chi_A' + \left(\frac{V_0^2}{r^{2p}} - \frac{m(p+m)}{r^2}\right)\chi_A = 0.$$

There is no dependence on the potential sign!

$\boxed{m \geqslant 0}$ $$\chi_A(r) \sim \frac{c_2}{r^{m+p}}, \quad \chi_B(r) \sim \frac{ic_2}{r^{m+1}}.$$

$\boxed{m \leqslant -1}$ $$\chi_A(r) \sim c_1 r^m, \quad \chi_B(r) \sim ic_1 r^{1+m-p}.$$

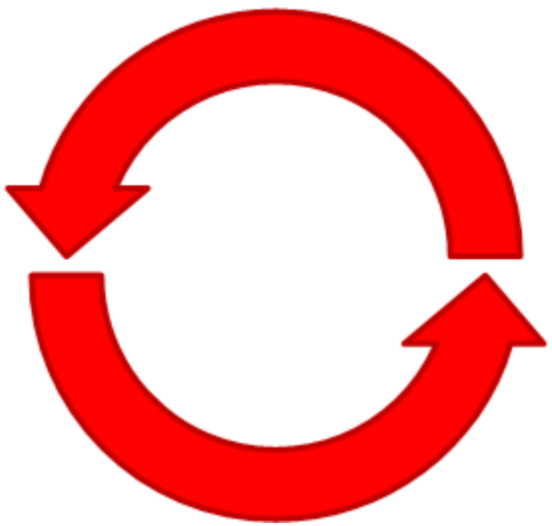
Fully-confined (square integrable) states

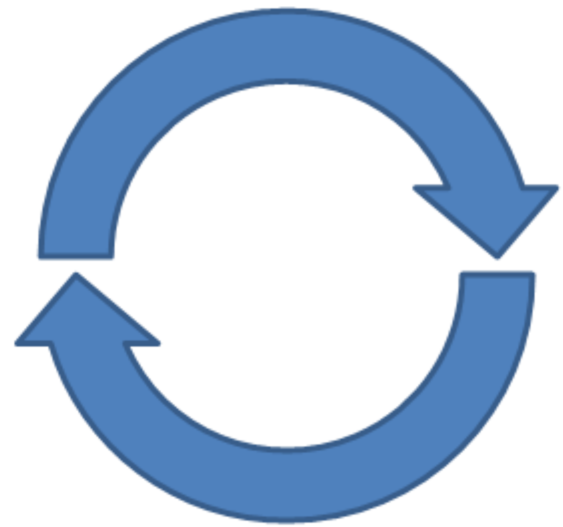
Fully-confined states for $E = 0$

=> DoS(0)≠0

Square integrable solutions require $m > 0$ or $m < -1$

=> vortices!





Recall: $C \equiv \frac{\boldsymbol{q}\hat{\boldsymbol{\sigma}}}{|\boldsymbol{q}|}, \ \langle C \rangle = \pm 1.$

Pseudo-spin is ill-defined for $E = 0$

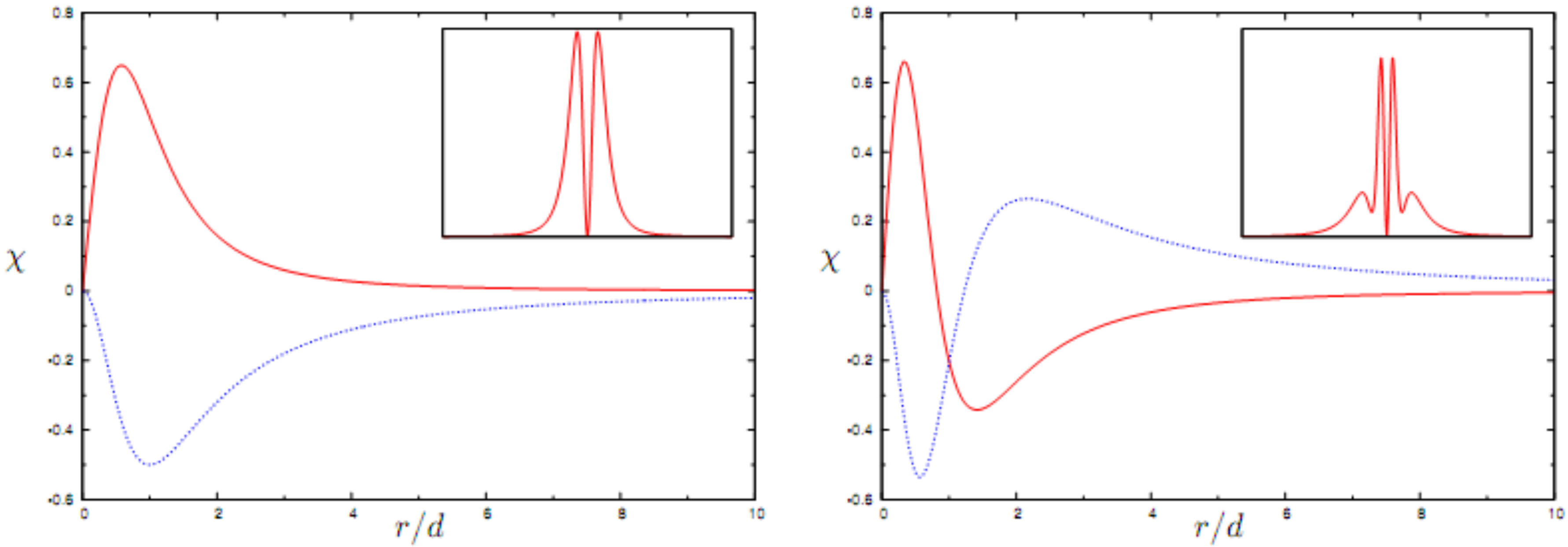
Exactly-solvable potential for $E = 0$

$$V(r) = \frac{V_0}{1 + (r/d)^2}$$

Condition for zero-energy states:

$$V_0 d = \begin{cases} 2(n + m + 1) & m > 0 \\ 2(n - m) & m < -1 \end{cases}$$

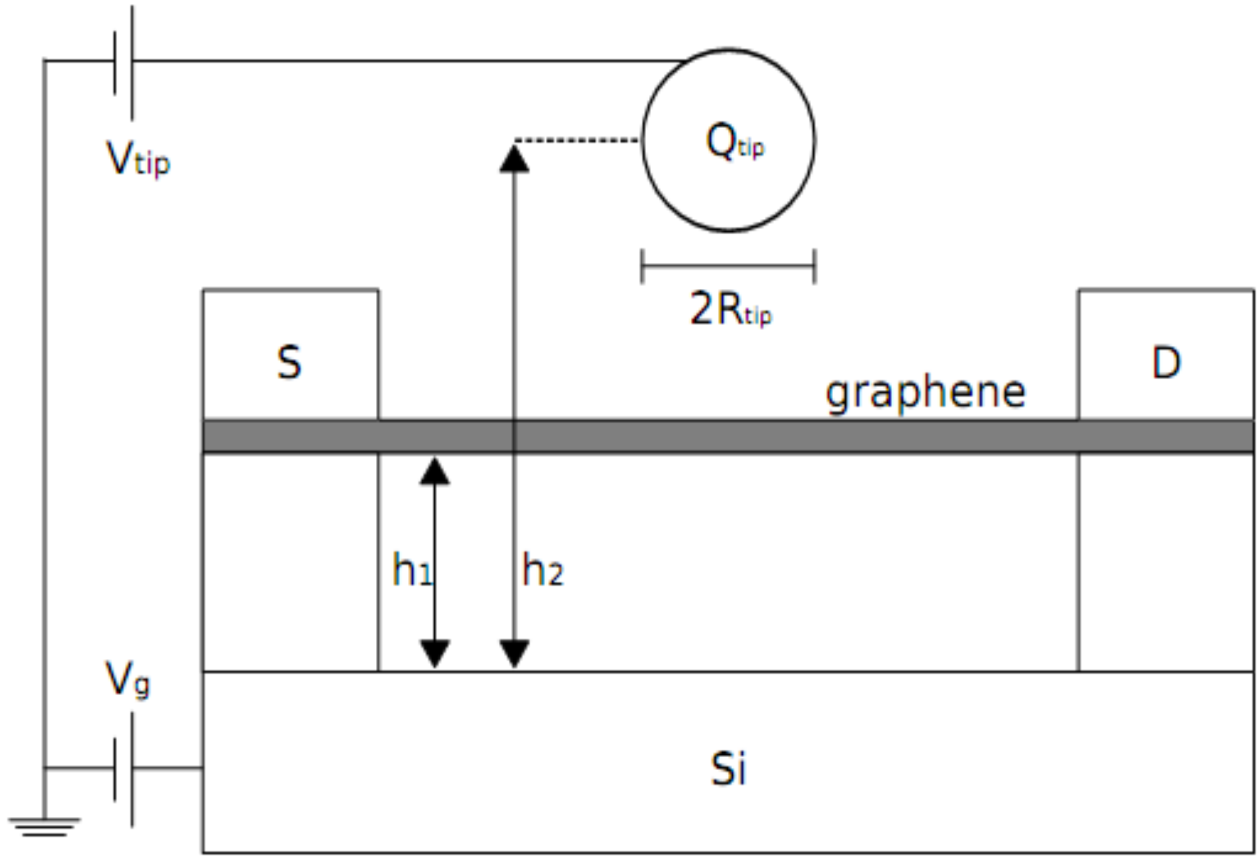
C.A.Downing, D.A.Stone & MEP, PRB **84**, 155437 (2011)



Wavefunction components and probability densities for the first two confined *m*=1 states in the Lorentzian potential

C.A.Downing, D.A.Stone & MEP, PRB **84**, 155437 (2011)

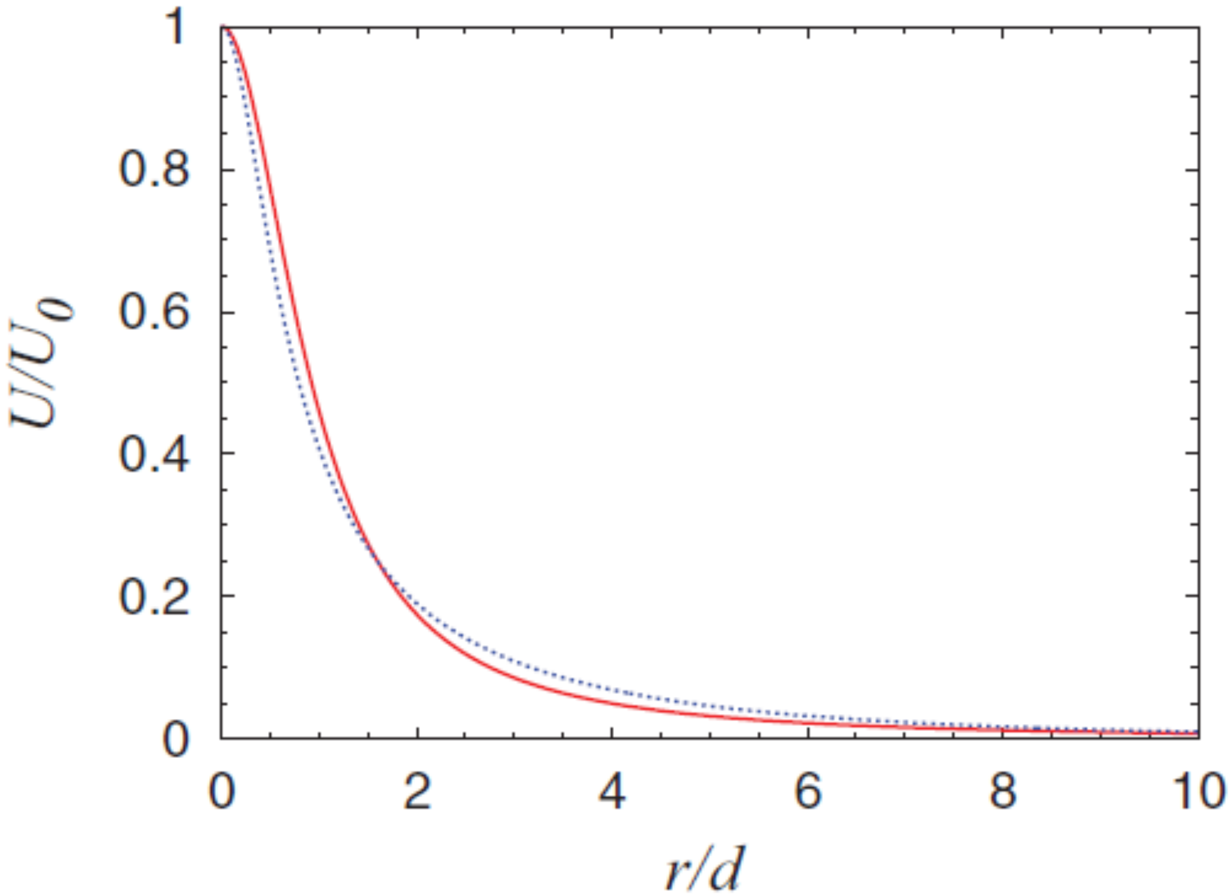
Relevance of the Lorentzian potential



STM tip above the graphene surface

STM tip above the graphene surface

$$U_{\mathrm{STM}}(r) \approx \frac{eQ_{\mathrm{tip}}}{4\pi\kappa}\left(\frac{1}{\sqrt{r^2+(h_1-h_2)^2}}-\frac{1}{\sqrt{r^2+(h_1+h_2)^2}}\right)$$

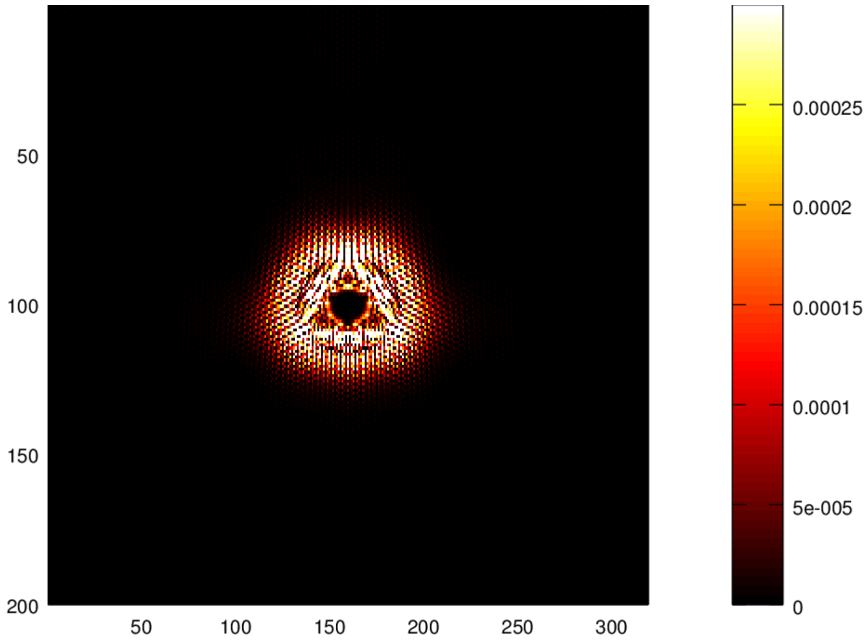


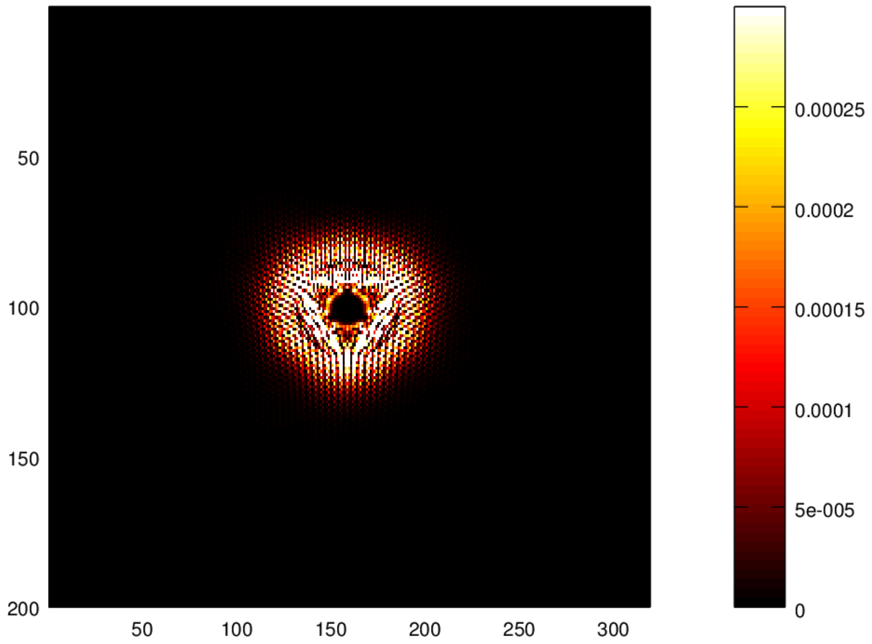
$$V_0 = \frac{eQ_{\mathrm{tip}}}{4\pi\kappa\hbar v_F}\frac{2h_1}{h_2^2-h_1^2}$$

$$d = \frac{h_2^2-h_1^2}{\pi h_1}\ln\left(\frac{h_2+h_1}{h_2-h_1}\right)$$

Coulomb impurity + image charge in a back-gated structure

Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)

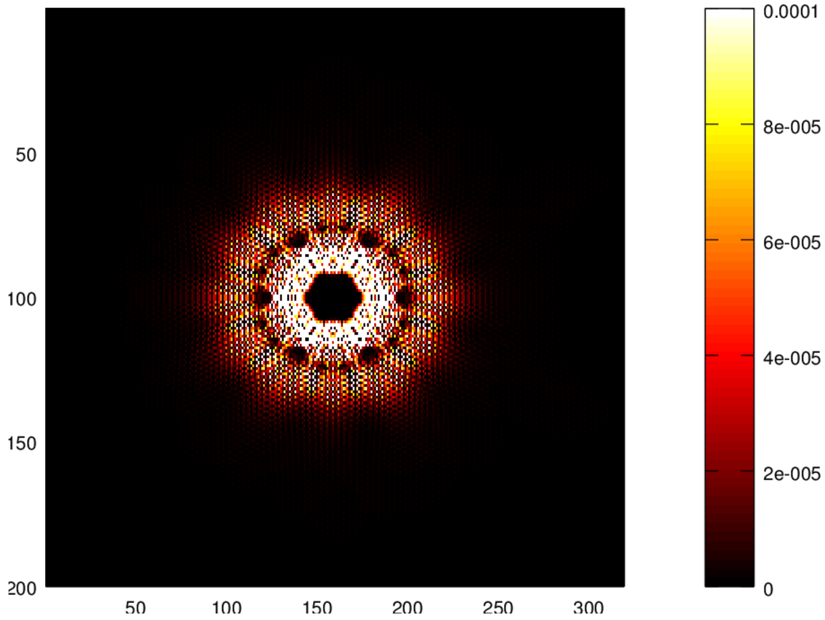


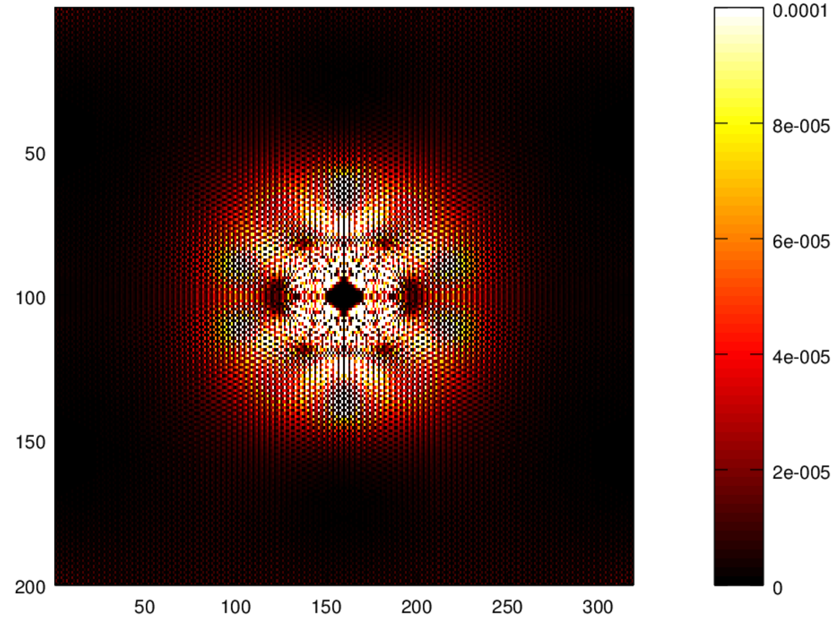


Potential is centred at an "A" atom.

Potential is centred at a "B" atom.

Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)

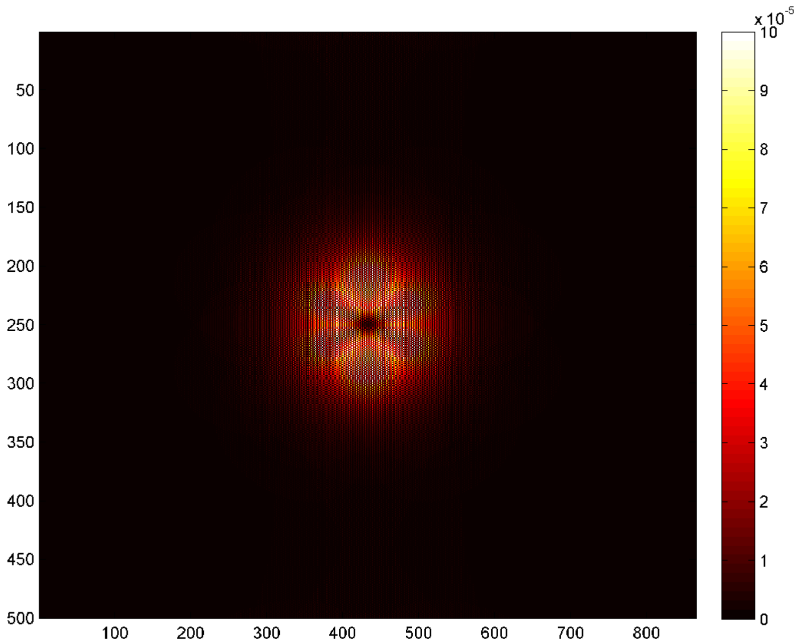


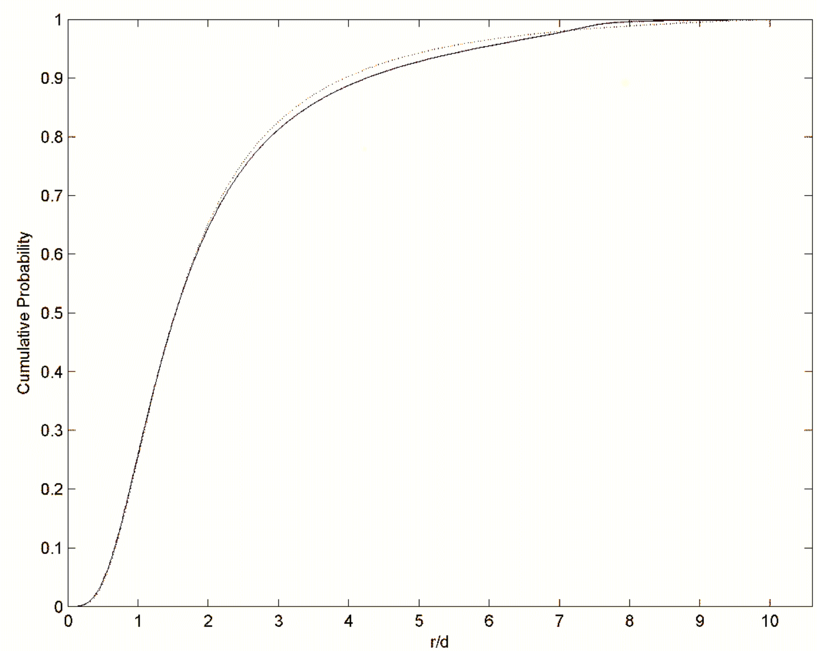


Potential is centred at the hexagon centre.

Potential is centred in the centre of a bond.

Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)





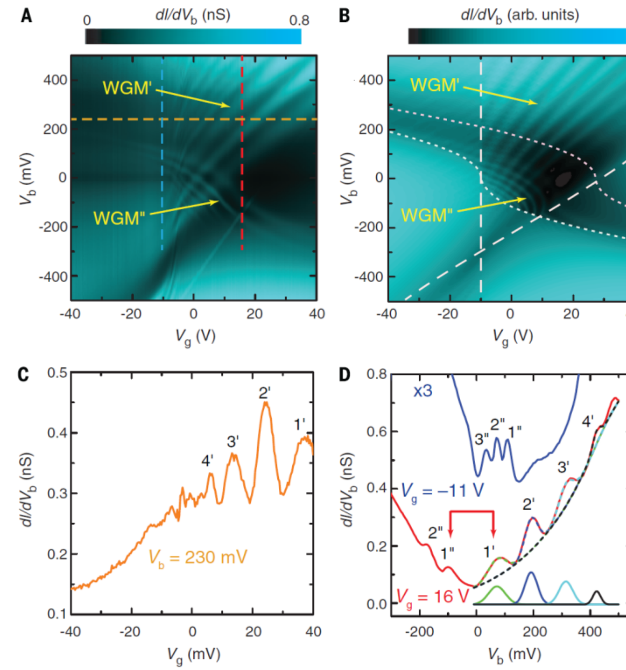
C.A. Downing, A. R. Pearce, R. J. Churchill, and MEP, PRB **92**, 165401 (2015)

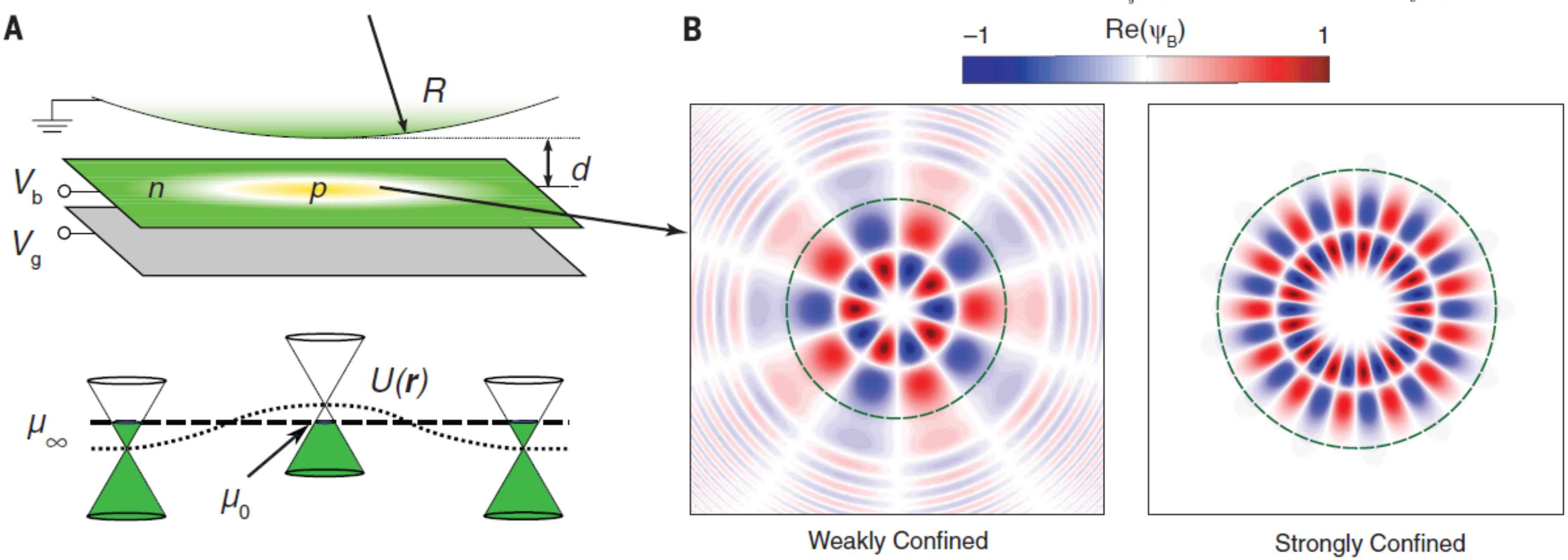
Experimental manifestations

Creating and probing electron whispering-gallery modes in graphene

Yue Zhao,[1,2]* Jonathan Wyrick,[1]* Fabian D. Natterer,[1]* Joaquin F. Rodriguez-Nieva,[3]* Cyprian Lewandowski,[4] Kenji Watanabe,[5] Takashi Taniguchi,[5] Leonid S. Levitov,[3] Nikolai B. Zhitenev,[1]† Joseph A. Stroscio[1]†

SCIENCE 672 8 MAY 2015 • VOL 348 ISSUE 6235





?? $\rho(r) \propto 1/(d + r^2/2R)$

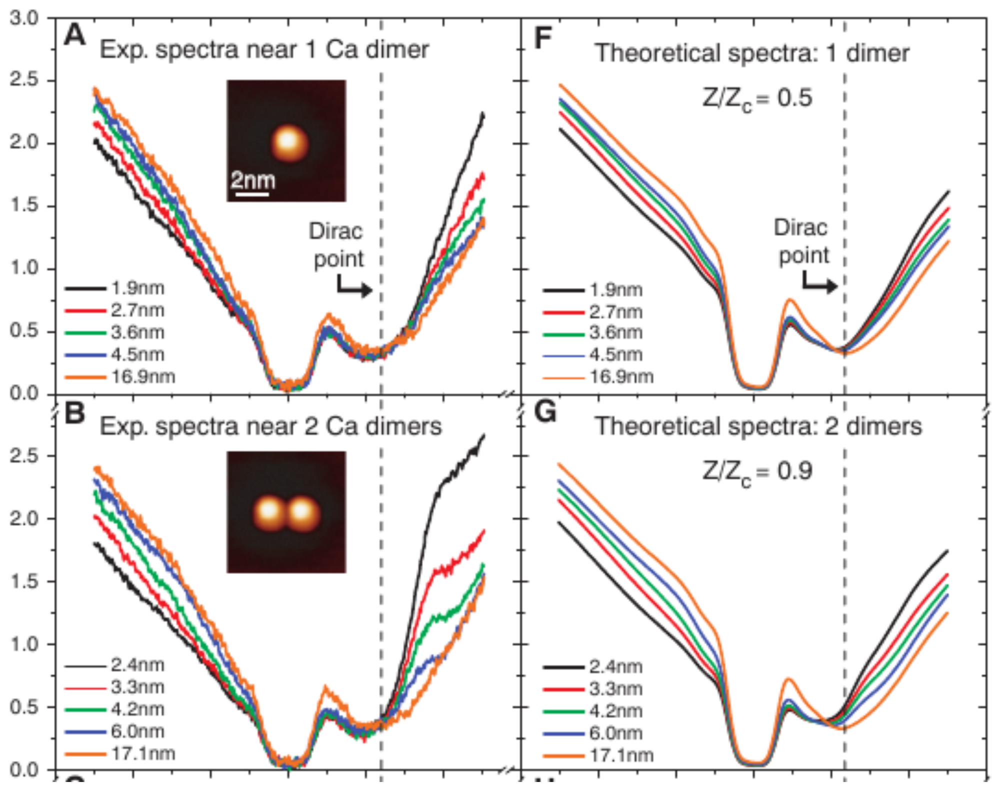
$$\varepsilon u(r) = \left(-i\sigma_x\partial_r + \frac{m+1/2}{r}\sigma_y + \kappa r^2\right)u(r)$$

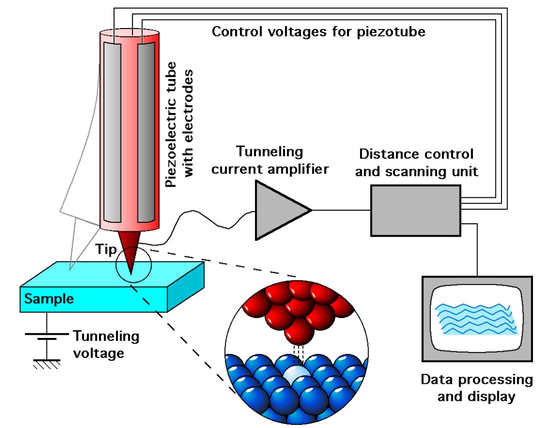
Crommie group experiments

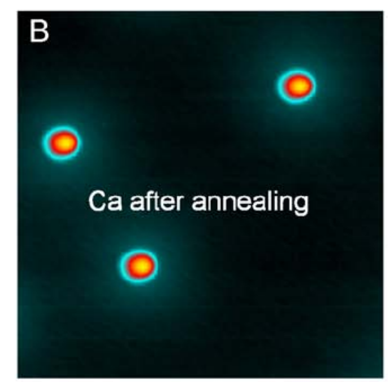
– Ca dimers on graphene have two states, charged and uncharged
– They can be moved around by STM tip, and the charge states can be manipulated
– Thus, one can make artificial atoms and study them via tunneling spectroscopy

Tunneling conductance (DOS)

$$\frac{dI}{dV}$$





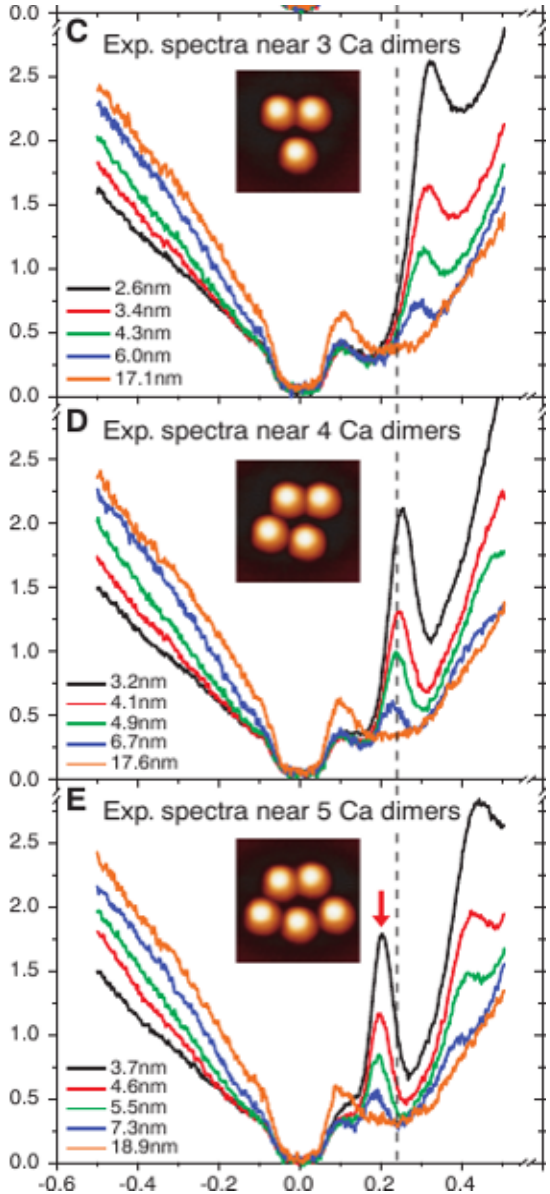


V

Tip-to-sample bias (electron energy)

Crommie group, Science **340**, 734 (2013) + "collapse" theory by Shytov and Levitov

Crommie group experiments vs atomic collapse theory



Electron density (Gate voltage)

Features **not** explained by the atomic collapse theory:

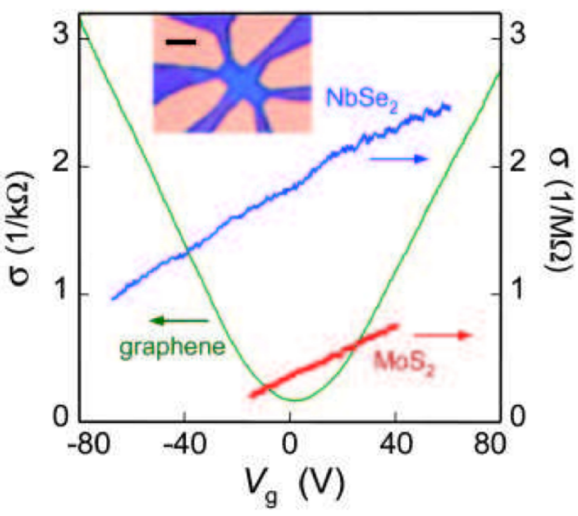
– The resonance is sensitive to doping.

– Sometimes it occurs on the “wrong” side with resect to the Dirac point.

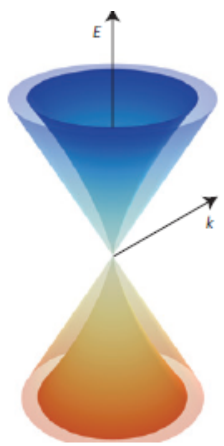
– There is a distance dependence of peak intensity.

Zero-energy states – So what?

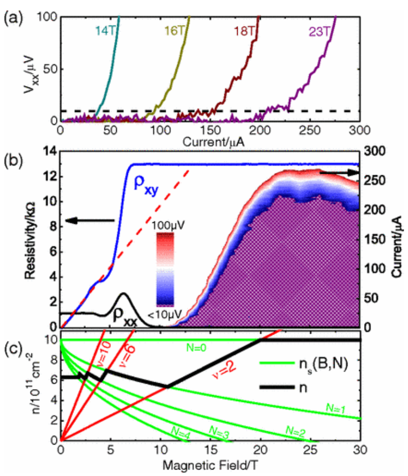
- Non-linear screening favors zero-energy states. Could they be a source of minimal conductivity in graphene for a certain type of disorder?
- Could the BEC of zero-energy bi-electron vortices provide an explanation for the Fermi velocity renormalization in gated graphene?
- Where do electrons come from in the low-density QHE experiments?



Novoselov et al., PNAS 102, 10451 (2005)



Elias et al., Nature Physics 7, 201 (2011)



Alexander-Webber et al., PRL 111, 096601 (2013)

A puzzle of excitons in graphene

Excitonic gap & ghost insulator (selected papers):

D. V. Khveshchenko, PRL. 87, 246802 (2001).

J.E. Drut & T.A. Lähde, PRL. 102, 026802 (2009).

T. Stroucken, J.H.Grönqvist & S.W.Koch, PRB 84, 205445 (2011).

and many-many others (Guinea, Lozovik, Berman etc…)

gap → mass → gap

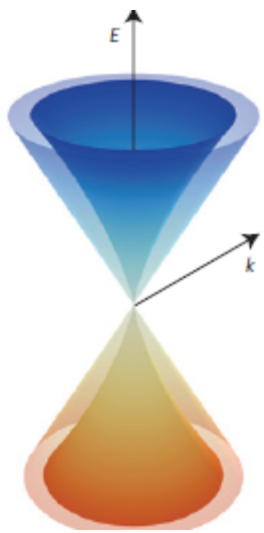
Warping => angular mass: Entin (e-h, K≠0), Shytov (e-e, K=0)

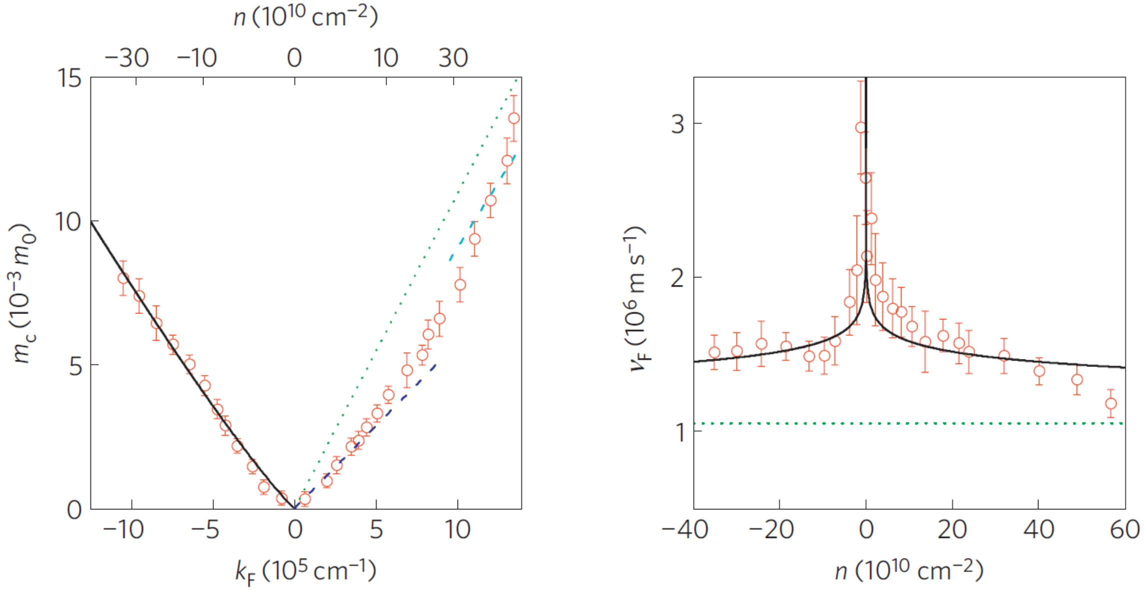
Massless particles do not bind! Or do they?

Binding at E=0 does not depend on the potential sign

Excitonic gap has never been observed!

Experiment: Fermi velocity renormalization...

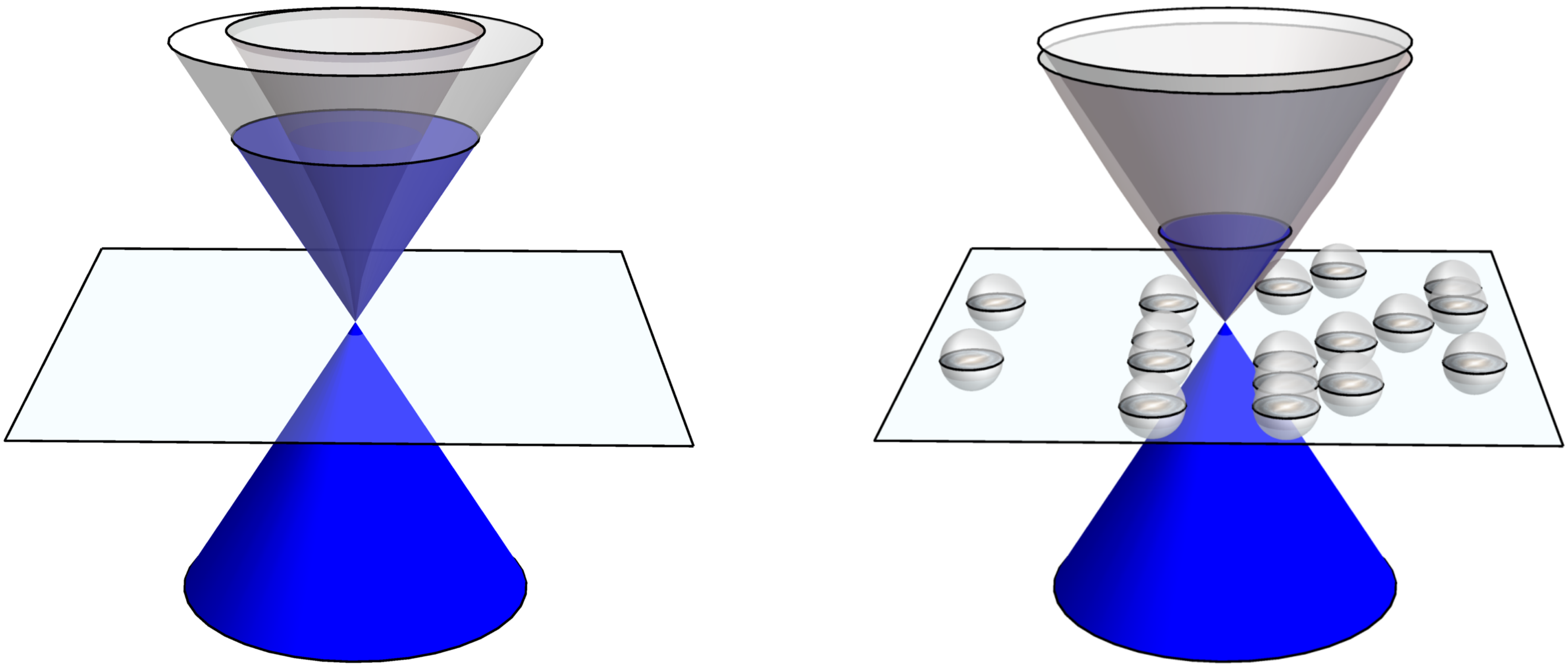




$$m_c = \hbar(\pi n)^{1/2}/v_F^* \quad ?? \quad v_F(n) = v_F(n_0)\left[1 + \frac{\alpha}{8\varepsilon_G}\ln(n_0/n)\right]$$

Elias, …, Geim, Nature Physics 7, 201 (2011)
Mayorov et al., Nano Lett. 12, 4629 (2012)

$$\frac{\sqrt{n}}{v_F^*} \text{ or } \frac{\sqrt{n^*}}{v_F} ?$$



Two-body problem – construction

Construct wavefunction

$$\begin{pmatrix} \psi_{A,A}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{A,B}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{B,A}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{B,B}(\mathbf{r_1},\mathbf{r_2}) \end{pmatrix}$$

Electron-hole

$$H_{e-h} = v_F \begin{bmatrix} V(r) & p_{x_e} - ip_{y_e} & -p_{x_h} + ip_{y_h} & 0 \\ p_{x_e} + ip_{y_e} & V(r) & 0 & -p_{x_h} + ip_{y_h} \\ -p_{x_h} - ip_{y_h} & 0 & V(r) & p_{x_e} - ip_{y_e} \\ 0 & -p_{x_h} - ip_{y_h} & p_{x_e} + ip_{y_e} & V(r) \end{bmatrix}$$

Electron-electron

$$H_{e1-e2} = v_F \begin{bmatrix} V(r) & p_{x_{e1}} - ip_{y_{e1}} & p_{x_{e2}} - ip_{y_{e2}} & 0 \\ p_{x_{e1}} + ip_{y_{e1}} & V(r) & 0 & p_{x_{e2}} - ip_{y_{e2}} \\ p_{x_{e2}} + ip_{y_{e2}} & 0 & V(r) & p_{x_{e1}} - ip_{y_{e1}} \\ 0 & p_{x_{e2}} + ip_{y_{e2}} & p_{x_{e1}} + ip_{y_{e1}} & V(r) \end{bmatrix}$$

Two body problem – free solutions

Diagonalize

$$E = \pm v_F \left(p_{x_1}^2 + p_{y_1}^2\right)^{1/2} \pm v_F \left(p_{x_2}^2 + p_{y_2}^2\right)^{1/2}$$

Centre-of-mass (COM) and relative coordinates

$$X = (x_1 + x_2)/2 \qquad Y = (y_1 + y_2)/2$$
$$x = x_1 - x_2 \qquad y = y_1 - y_2$$

So equivalently

$$E/\hbar v_F = \pm\left((K_X/2 + k_x)^2 + (K_Y/2 + k_y)^2\right)^{1/2} \pm \left((K_X/2 - k_x)^2 + (K_Y/2 - k_y)^2\right)^{1/2}$$

COM and relative ansatz

$$\Psi_i(\boldsymbol{R}, \boldsymbol{r}) = \exp(i\boldsymbol{K}\cdot\boldsymbol{R})\psi_i(\mathbf{r})$$

COM momentum K=0,
system reduces to 3 by 3 matrix

$$\begin{bmatrix} \frac{U(r)-E}{\hbar v_F} & \partial_r + \frac{m}{r} & 0 \\ 2\left(-\partial_r + \frac{m-1}{r}\right) & \frac{U(r)-E}{\hbar v_F} & -2\left(\partial_r + \frac{m+1}{r}\right) \\ 0 & \partial_r - \frac{m}{r} & \frac{U(r)-E}{\hbar v_F} \end{bmatrix} \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_4(r) \end{bmatrix} = 0, \quad (4)$$

Two body problem – bound states

$$\begin{bmatrix} \frac{U(r)-E}{\hbar v_F} & \partial_r + \frac{m}{r} & 0 \\ 2\left(-\partial_r + \frac{m-1}{r}\right) & \frac{U(r)-E}{\hbar v_F} & -2\left(\partial_r + \frac{m+1}{r}\right) \\ 0 & \partial_r - \frac{m}{r} & \frac{U(r)-E}{\hbar v_F} \end{bmatrix} \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_4(r) \end{bmatrix} = 0, \quad (4)$$

Only binding at Dirac point energy E=0, consider interaction potential

$$U(r) = -U_0/(1+(r/d)^2)$$

$$\phi_2(r) = \frac{A}{d} \times \frac{(r/d)^{|m|}}{(1+(r/d)^2)^\eta} f(r),$$

Angular momentum

$$m = 0, \pm 1, \pm 2, ...$$

Gauss hypergeometric

$$f(\xi) = {}_2F_1\left(-n, -n + \frac{1}{2}\frac{U_0 d}{\hbar v_F}; |m|+1; \frac{\xi}{1+\xi}\right)$$

useful to define

$$\eta = \frac{|m| + 1 + \sqrt{|m|^2+1}}{2},$$

Two-body problem – exactly solvable model

$$\frac{U_0 d}{\hbar v_F} = \frac{300}{137}\frac{1}{\epsilon}\frac{d}{r_0} = 4(n+\eta), \quad n = 0, 1, 2...$$

$$\eta = \frac{|m| + 1 + \sqrt{|m|^2 + 1}}{2}$$

1 - Length scale d of the order of 30 nm due to necessity of gate
2- Cut-off energy depends on geometry and differs strongly for monolayer graphene or interlayer exciton in spatially separated graphene layers
3-Results do not depend on the sign of the interaction potential

<u>1. Monolayer vortex</u>
Cut-off comes from Ohno strength of 11.3 eV, thus r0 = 0.064nm

$$U_0 = \frac{e^2}{4\pi\epsilon_0\epsilon} r_0^{-1}$$

<u>2. Interlayer exciton</u>
Cut-off comes from interlayer spacing of r0 = 1.4nm

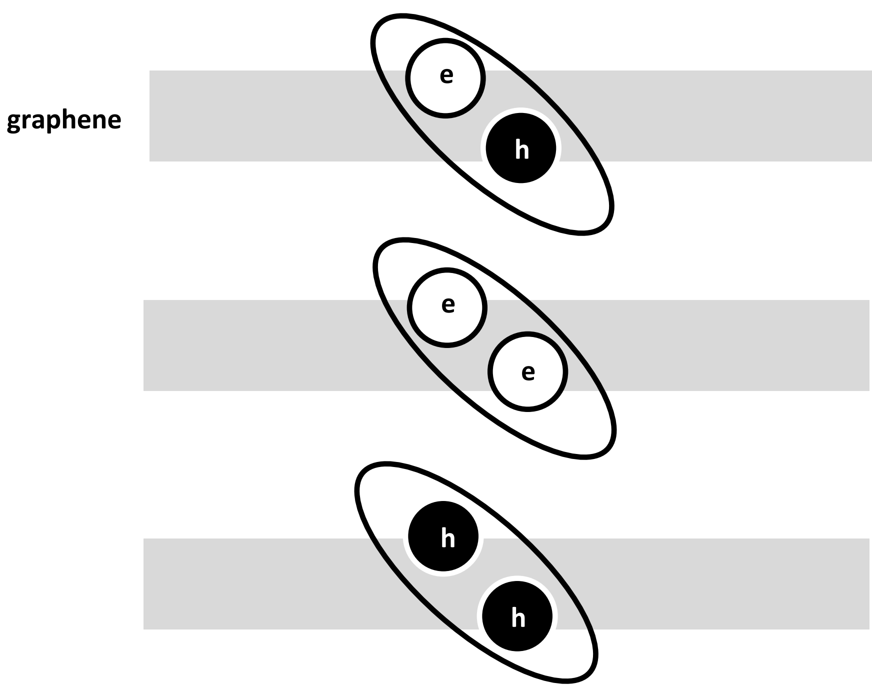
Nb assuming BN with relative permittivity of ϵ = 3.2

Exactly solvable model – two systems

1. Monolayer exciton or e-e pair

U0d = 515.39…

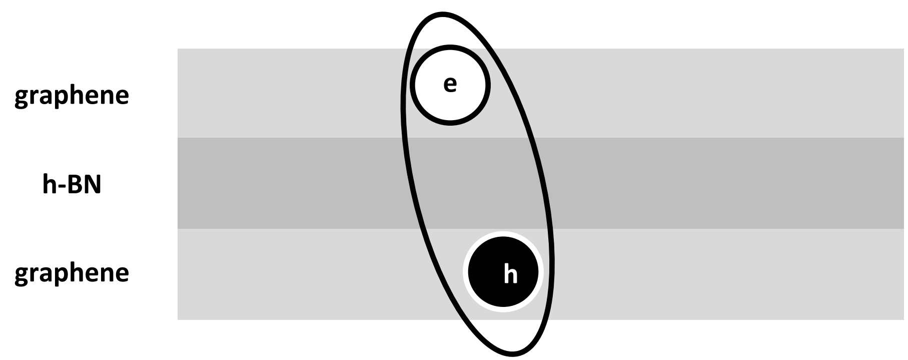
(m, n) = (128, 0), size <r> = 1.006 d
(m, n) = (127, 1), size <r> = 1.018 d
(m, n) = (126, 2), size <r> = 1.030 d



2. Interlayer exciton

U0d = 14.66…

(m, n) = (3, 0), size <r> = 1.433 d
(m, n) = (2, 1), size <r> = 2.639 d
(m, n) = (1, 2), size <r> = 4.415 d



Two-body problem – exactly solvable model

Results for d =100 nm, monolayer graphene, repulsive interaction

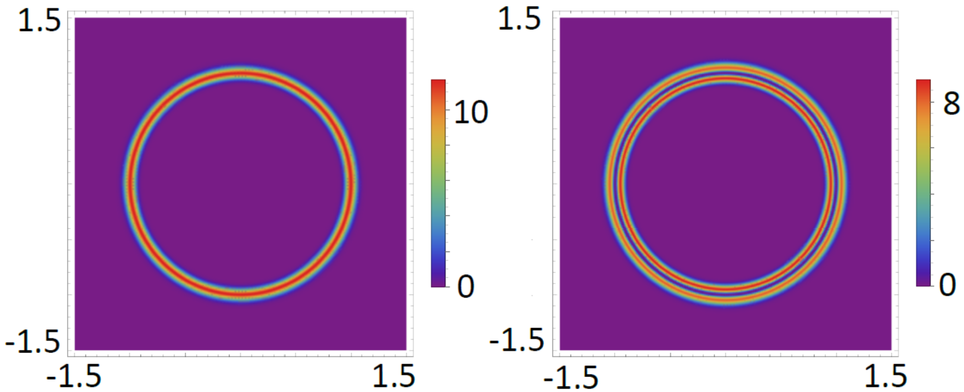
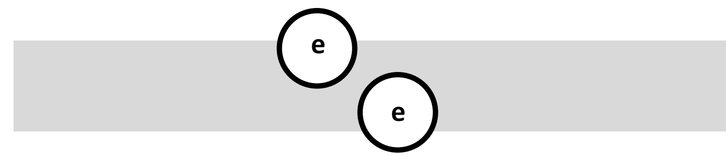
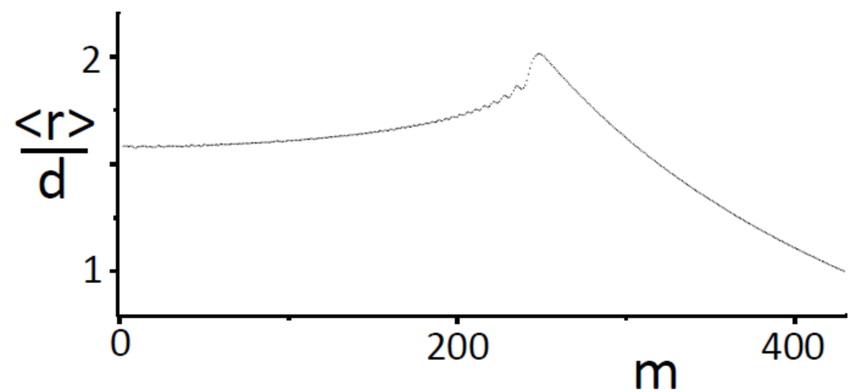


FIG. 1: Radial probability densities for the pair states with quantum numbers $(n, m) = (0, 429), (1.428)$ left-to-right.





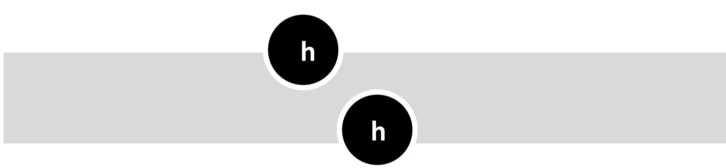


FIG. 2: A plot of the average size of the pair state as a function of quantum number m.

C.A.Downing & MEP, arXiv: 1506.04425.

Static screening

REVIEWS OF MODERN PHYSICS, VOLUME 83, APRIL–JUNE 2011

Electronic transport in two-dimensional graphene

S. Das Sarma et al.

TABLE I. Elementary electronic quantities. Here E_F, $D(E)$, r_s, and $q_{\rm TF}$ represent the Fermi energy, the density of states, the interaction parameter, and the Thomas-Fermi wave vector, respectively. $D_0 = D(E_F)$ is the density of states at the Fermi energy and $q_s = q_{\rm TF}/k_F$.

	E_F	$D(E)$	$D_0 = D(E_F)$	r_s	$q_{\rm TF}$	q_s
MLG	$\hbar v_F\sqrt{\frac{4\pi n}{g_s g_v}}$	$\frac{g_s g_v E}{2\pi(\hbar v_F)^2}$	$\frac{\sqrt{g_s g_v n}}{\sqrt{\pi}\hbar v_F}$	$\frac{e^2}{\kappa\hbar v_F}\ \frac{\sqrt{g_s g_v}}{2}$	$\frac{\sqrt{4\pi g_s g_v n e^2}}{\kappa\hbar v_F}$	$\frac{g_s g_v e^2}{\kappa\hbar v_F}$
BLG and 2DEG	$\frac{2\pi\hbar^2 n}{m g_s g_v}$	$\frac{g_s g_v m}{2\pi\hbar^2}$	$\frac{g_s g_v m}{2\pi\hbar^2}$	$\frac{me^2}{2\kappa\hbar^2}\ \frac{g_s g_v}{\sqrt{\pi n}}$	$\frac{g_s g_v m e^2}{\kappa\hbar^2}$	$\frac{(g_s g_v)^{3/2} m e^2}{\kappa\hbar^2\sqrt{4\pi n}}$

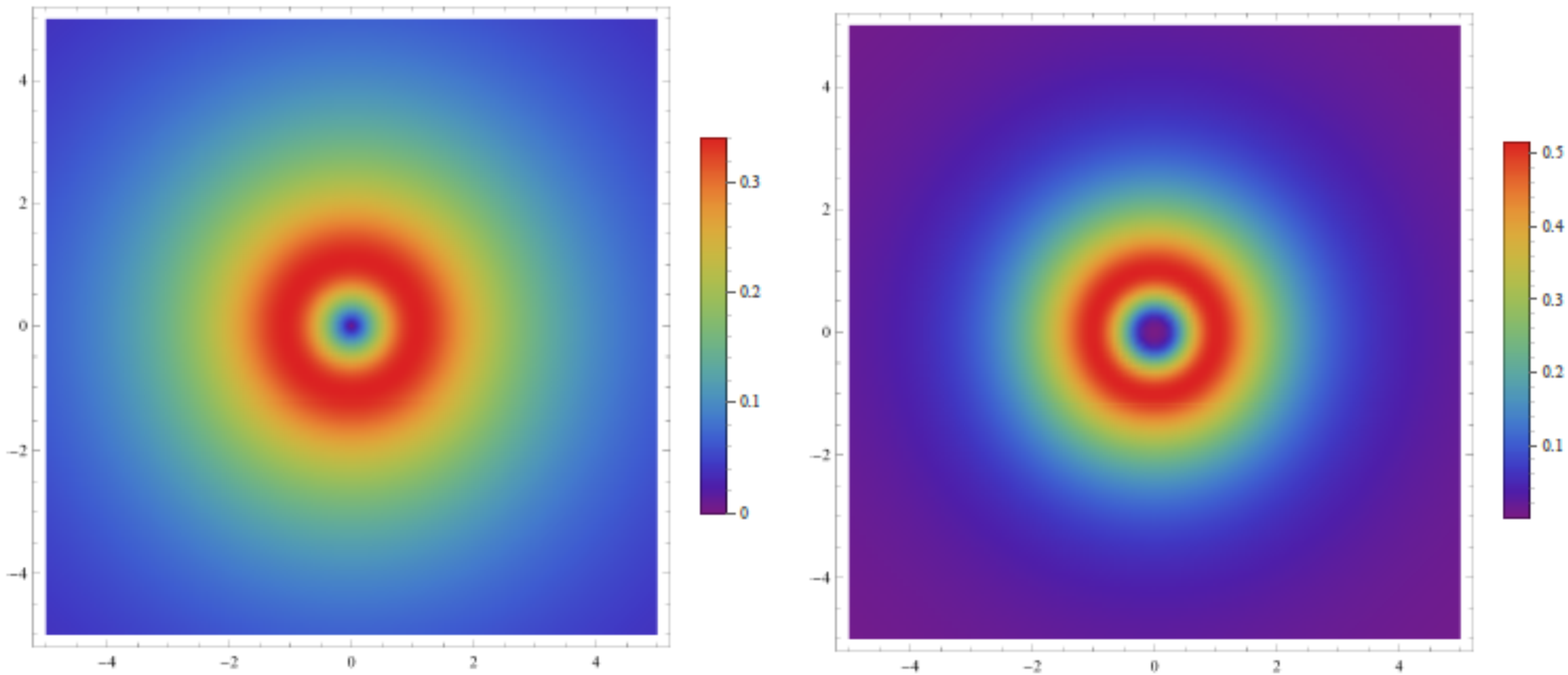
$$V_q \sim \frac{2\pi}{q + q_{\rm TF}}.$$

$$V(r) \approx \frac{\alpha}{\sqrt{r^2 + r_0^2(1 + q_{\rm TF}^2 r^2)}}.$$

Gate-induced image charge

$$V(r) = \alpha\left(\frac{1}{\sqrt{r^2 + r_0^2}} - \frac{1}{\sqrt{r^2 + d^2}}\right)$$

Two-body problem – exactly solvable model



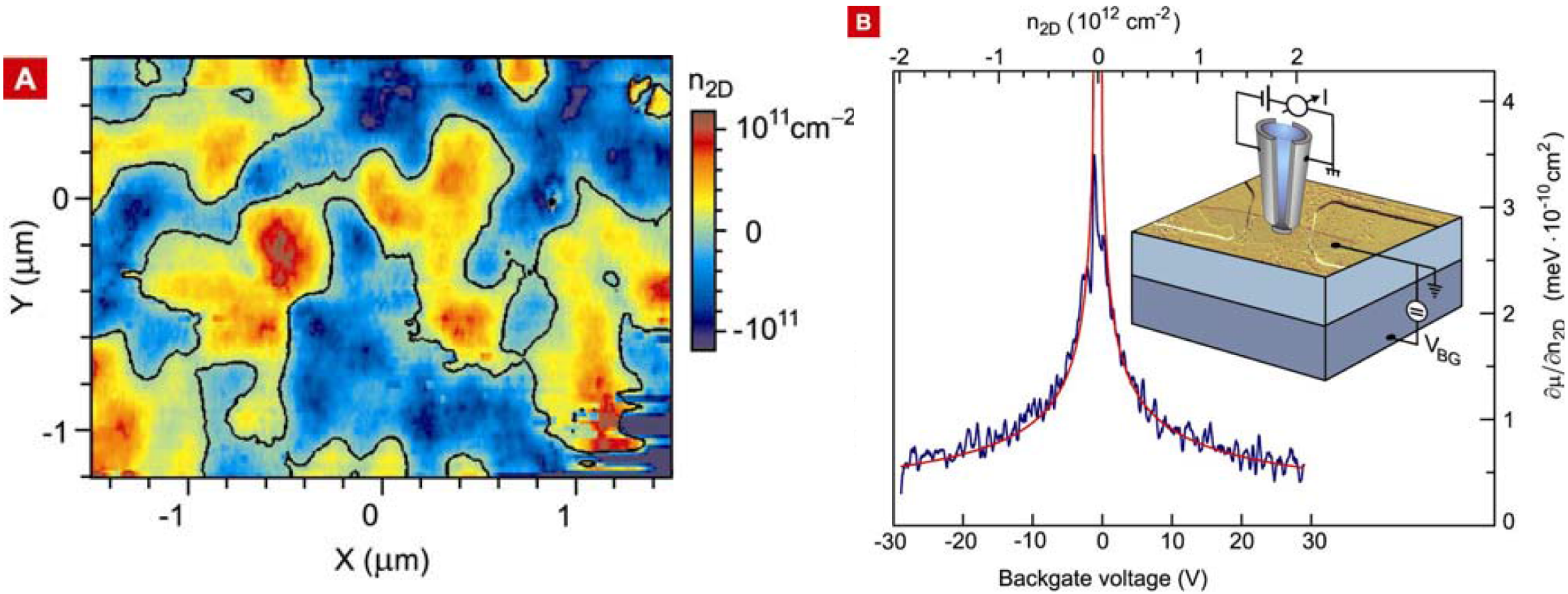
Radial probability densities for lowest pair states with quantum numbers $(m, n) = (1, 0), (2, 0)$

$r \sim 0,\ \ \phi_2 \sim r^{|m|}$

$r \to \infty \qquad \phi_2 \sim r^{|m|-2\eta},$

$$\eta = \frac{|m|+1+\sqrt{m^2+1}}{2}$$

Electron-hole puddles in disordered graphene or droplets of two-particle vortices?



J. Martin, N. Akerman, G. Ulbricht, T. Lohmann, J. H. Smet, K. von Klitzing & A. Yakobi, *Observation of electron–hole puddles in graphene using a scanning single-electron transistor*, Nature Physics **4**, 144 (2008)
[cited by over a 1000 times]

QHE experiments

PRL 111, 096601 (2013) PHYSICAL REVIEW LETTERS week endi
30 AUGUST

Phase Space for the Breakdown of the Quantum Hall Effect in Epitaxial Graphene

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(Received 17 April 2013; published 27 August 2013)

We report the phase space defined by the quantum Hall effect breakdown in polymer gated epitaxial graphene on SiC (SiC/G) as a function of temperature, current, carrier density, and magnetic fields up to 30 T. At 2 K, breakdown currents (I_c) almost 2 orders of magnitude greater than in GaAs devices are observed. The phase boundary of the dissipationless state ($\rho_{xx} = 0$) shows a $[1 - (T/T_c)^2]$ dependence and persists up to $T_c > 45$ K at 29 T. With magnetic field I_c was found to increase $\propto B^{3/2}$ and $T_c \propto B^2$. As the Fermi energy approaches the Dirac point, the $\nu = 2$ quantized Hall plateau appears continuously from fields as low as 1 T up to at least 19 T due to a strong magnetic field dependence of the carrier density.

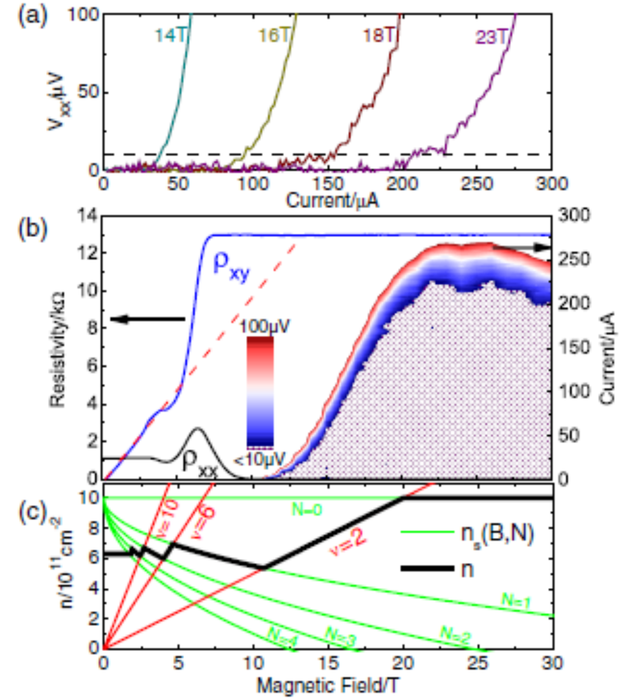


FIG. 1 (color online). (a) $I - V_{xx}$ characteristics of sample 1 at 2.0 K, with a breakdown condition of $V_{xx} = 10\ \mu V$, giving a maximum critical current density $j_c = 43$ A/m at 23 T. (b) Combined magnetotransport [ρ_{xy} and ρ_{xx}] data and $I - V_{xx} - B$ contour plot; the hashed region represents $V_{xx} < 10\ \mu V$, the dissipationless quantum Hall regime. Extrapolating the low field Hall coefficient to $\rho_{xy} = h/2e^2$ (dashed red line) gives the expected field for the peak breakdown of $\nu = 2$ without a field dependent n. (c) Magnetic field dependence of the carrier density (thick black line), following lines of constant filling factor (thin red lines) while E_F lies between Landau levels and then the charge transferred from surface donors in SiC, $n(B, N)$ (thin green curves), while the Landau levels fill, from the model in Ref. [7].

Nicholas group, PRL 111, 096601 (2013)

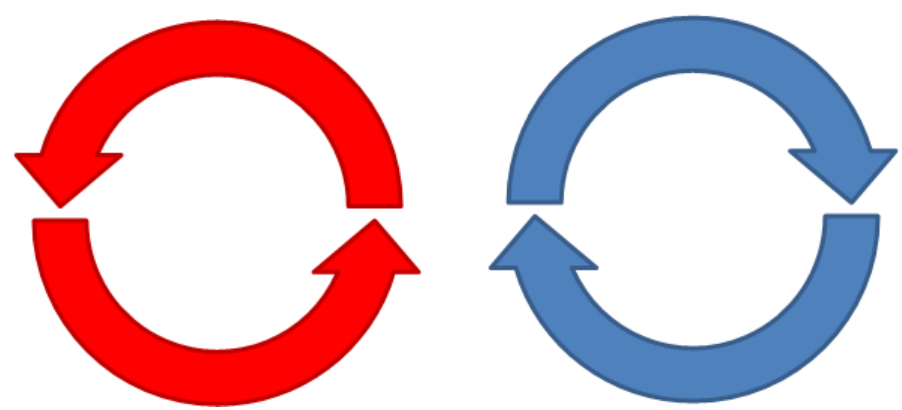
Also seen by many other groups:
Janssen et. al, PRB **83**, 233402 (2011)
Ribeiro-Palau, Nature Comm. (2015)
Benoit Jouault (2011-2015)
Agrinskaya (2013) etc…

Apparent difference in carrier densities without B and in a strong magnetic field.
Reservoir of “silent” carriers?

Is it a step in the on-going search for Majorana fermions in condensed matter systems?

Ettore Majorana
1906 - ?

"Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense." (E. Fermi)



Practical applications?

APPLIED PHYSICS LETTERS **104**, 161116 (2014)

Graphene—A rather ordinary nonlinear optical material

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Essentially, all the electrons residing further away from the Dirac point hardly contribute to nonlinearity at all, but they are needed to keep the Fermi level high enough to mitigate the loss due to band-to-band absorption. Clearly, absence of a real bandgap (rather than one induced by blocking the absorption) is a handicap that seems to have no remedy.

Remedy – a reservoir of charged vortices at the Dirac point.

Highlights

●Contrary to the widespread belief electrostatic confinement in graphene and other 2D Weyl semimetals is indeed possible!

● Several smooth fast-decaying potential have been solved exactly for the 2D Dirac-Weyl Hamiltonian.

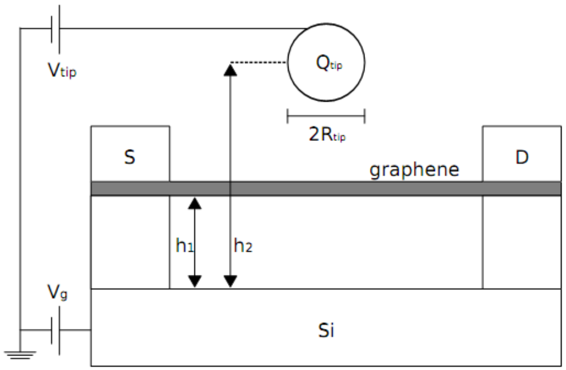
● Precisely tailored potentials support zero-energy states with non-zero values of angular momentum (vortices). The threshold in the effective potential strength is needed for the vortex formation.

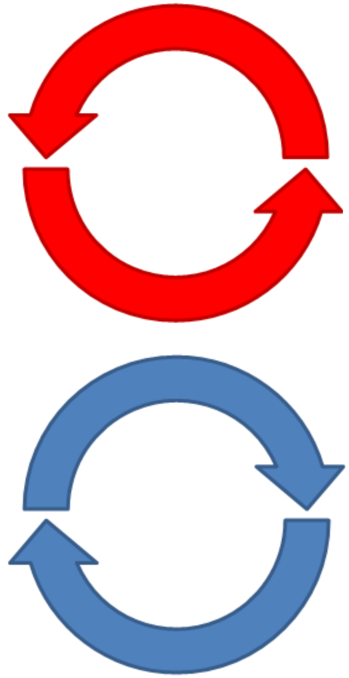
● An electron and hole or two electrons (holes) can also bind into a zero-energy vortex reducing the total energy of the system.

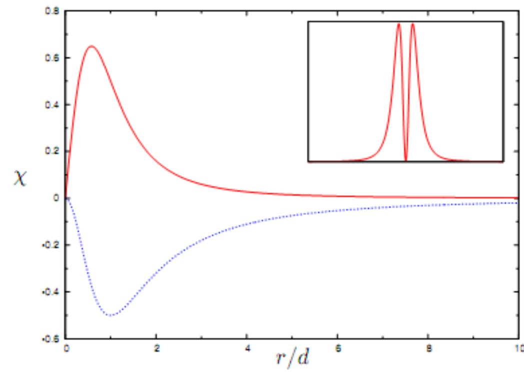
● The existence of zero-energy vortices explains several puzzling experimental results in gated graphene.

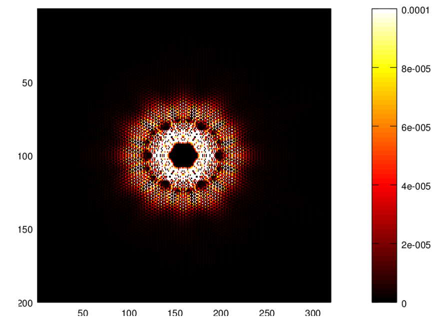
● Confined modes might also play a part in minimum conductivity (puddles)?

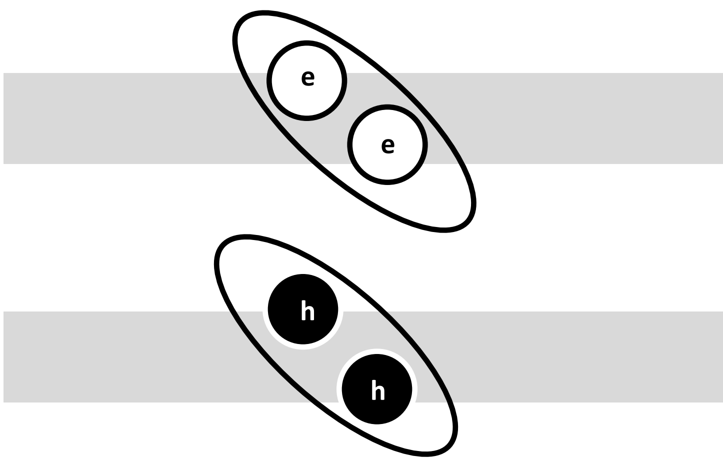
Highlights

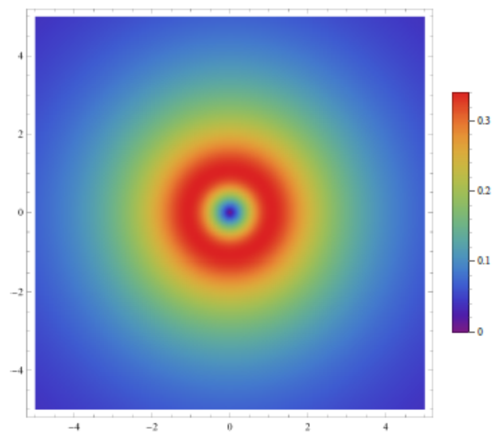


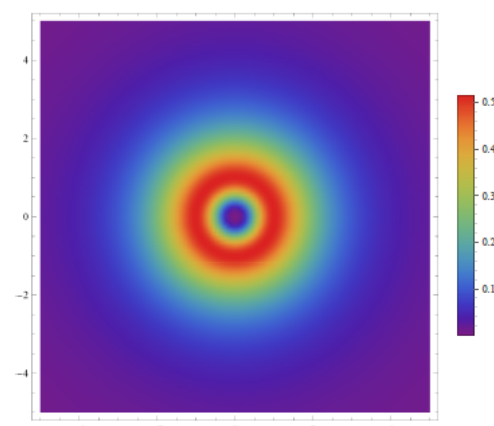












Thank you for your attention!