Fano Blockade by a Bose-Einstein Condensate in an Optical Lattice

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We study the transport of atoms across a localized Bose-Einstein condensate in a one-dimensional optical lattice. For atoms scattering off the condensate, we predict total reflection as well as full transmission for certain parameter values on the basis of an exactly solvable model. The findings of analytical and numerical calculations are interpreted by a tunable Fano-like resonance and may lead to interesting applications for blocking and filtering atom beams.

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An understanding of the transport properties of ultracold atoms is vital for the development of technological applications in the fields of matter-wave interferometry [1] or quantum information processing with neutral atoms [2–5]. Over the last couple of years, it has been shown that optical lattices, generated by counter-propagating laser beams and providing a periodic potential modulation for the atoms, introduce many interesting and potentially useful effects by modifying single atom properties and enhancing correlations between atoms [6]. Here, we discuss the scattering of atoms across a localized Bose-Einstein condensate (BEC) in an optical lattice. We find that dramatic effects of scattering resonances with either full transparency or total reflection can occur.

Previously, transparency effects have been conjectured for the scattering of He atoms on a film of superfluid helium-4 [7], and similar effects have been predicted for the scattering of atoms on a BEC in a trap of finite depth [8]. These effects were first attributed to the coherent interactions within the target and with the scattering atoms, but a full understanding of the numerical results was not achieved. More numerical results were produced later [9] and a Levinson theorem was proved on general grounds [10] without revealing the mechanism for transparency effects.

In this Letter, we present and analyze a very simple and analytically solvable one-dimensional model of atom scattering by a BEC. The model shows transparency as well as blockade of atoms by total reflection, which is interpreted as a Fano resonance. Fano originally studied how interference may both enhance and suppress scattering close to a Feshbach resonance [11,12]. In the problem considered here, the atom-atom interaction leads to an effective nonlinearity. It was shown recently [13] that nonlinearity generates several scattering channels, which can lead to resonances with destructive interference and in 1D, particularly, to total reflection similar to the original Fano problem. Hence, they are termed *Fano resonances*. Proposed applications in nonlinear optics and Josephson junction networks encounter various difficulties due to inhomogeneities and dissipation [14]. They are absent in the present study, thus making the resonant atom-BEC scattering ideal for harvesting on Fano resonances.

We consider a BEC on a lattice, where interactions between atoms are present only in a very localized region [see Fig. 1]. Such a situation could be realized experimentally by combining optical lattices with atom-chip technology [15,16] or in optical micro-lens arrays [17] where the s-wave scattering length of atoms can be tuned by an inhomogeneous magnetic [18,19] or laser field [20,21]. Specifically, we consider the discrete nonlinear Schrödinger (DNLS) equation, a classical variant of the Bose-Hubbard model appropriate for a BEC in a periodic potential in the tight binding limit [6]. With interactions being present only on site number n_c, we write in dimensionless form

$$i\frac{\partial \Psi_n}{\partial t} = -(\Psi_{n+1} + \Psi_{n-1}) - \gamma|\Psi_{n_c}|^2\Psi_{n_c}\delta_{n,n_c}, \qquad (1)$$

where $\Psi_n(t)$ is a complex amplitude of the BEC field at site n and $-\gamma = U/J$ is the interaction strength on site n_c. This simple model reflects generic features of BECs in a one-

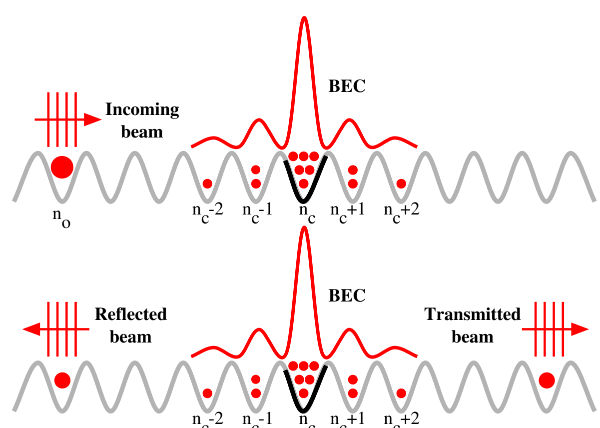

FIG. 1 (color online). Scattering scheme in an optical lattice. The incoming, reflected, and transmitted beams of atoms are represented as plane waves. The atoms interact only around $n = n_c$, where the BEC is centered.

dimensional optical lattice with inhomogeneous scattering length. Furthermore, this model could be realized quantitatively in a deep optical lattice with tight transverse confinement [22]. For atoms with mass M in a lattice with spacing d in the tight binding limit, $J \approx 4s^{3/4}e^{-2\sqrt{s}}E_r/\sqrt{\pi}$ is the energy scale for tunneling between the lattice sites, where $s = V_0/E_r$ is the depth of the optical lattice V_0 measured in units of the recoil energy $E_r = 2\hbar\pi^2/(d^2M)$. The on-site interaction energy per atom is $U = 4\pi a_s\hbar^2 \int d^3x|\psi(\mathbf{x})|^4/M$, where a_s is the tunable s-wave scattering length at the nonlinear site n_c, and $\psi(\mathbf{x})$ is the localized Wannier state associated with the lowest Bloch band of the lattice. The number of atoms in the lattice is given by $N = \sum_n|\Psi_n|^2$. Small-amplitude plane-wave solutions of Eq. (1) take the form $\Psi_n = \Psi_0\exp(ikn)\exp(-iE_kt)$ and satisfy the relation

$$E_k \equiv -2\cos k, \tag{2}$$

which defines the band of single-particle energies $E_k \in [-2, 2]$ [see Fig. 2(a)]. The unit of dimensionless energy is J, and the quasimomentum k is measured in units of d^{-1}.

First, we look for localized and stationary solutions of Eq. (1), corresponding to the BEC centered in $n = n_c$. For simplicity, we assume that interactions are attractive and thus $\gamma > 0$. As an ansatz, we take an exponentially localized profile: $\Psi_n(t) = b_n(t) = b_{n_c}x^{|n-n_c|}\exp(-iE_bt)$, where b_{n_c} is the condensate amplitude, $|x| < 1$, and E_b is the respective energy or chemical potential. Inserting this expression into (1), we obtain that

$$E_b = -\sqrt{4 + g^2} \quad \text{and} \quad x = -(E_b + g)/2, \tag{3}$$

where $g \equiv \gamma b_{n_c}^2$ ($g > 0$). Equation (3) corresponds to solutions for localized BECs with E_b being outside of the band E_k [$E_b < -2$, see Fig. 2(a)]. They are similar to bright lattice solitons pinned to the nonlinear lattice site.

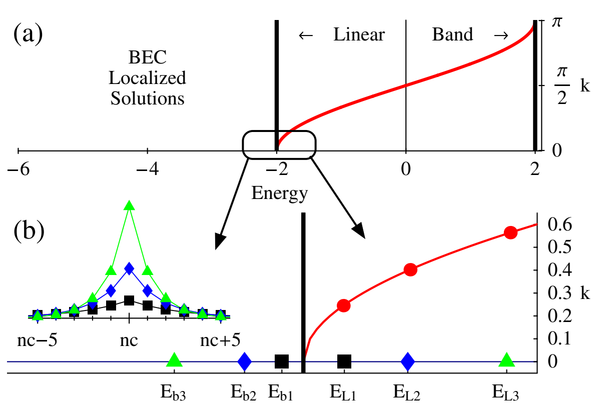

FIG. 2 (color online). (a) Energy diagram for localized and extended solutions. In the linear band, E_k is plotted. (b) Zoom of the region $E \sim -2$. Boxes, diamonds, and triangles correspond to the BEC ($E_b < -2$) and to the local mode ($E_L > -2$) solutions for g_1, g_2, and g_3, respectively. The corresponding BEC profiles are shown. The filled circles correspond to the resonance condition (12).

These localized modes exist only above a threshold $N_b = -E_b/\gamma > N_b^{\text{th}}$ [23], given by $N_b^{\text{th}} = 2/\gamma$. We assume that N_b is significantly larger than the threshold, which should be easily achieved in a possible experiment.

By controlling the number of atoms in the BEC or by tuning the nonlinear coefficient, we can easily modify the BEC energy $E_b = -\gamma N_b$ (this is equivalent to modifying the parameter g). This is one of the keys for a tunable Fano-blockade scheme. As we will show later, this energy is directly related to the energy where zero transmission of the atom beam through the BEC is observed.

We consider three different values for g as examples: $g_1 \equiv 0.36$, $g_2 \equiv 0.6$, and $g_3 \equiv 0.9$. The corresponding energies [$E_{b1} = -2.03$ (box), $E_{b2} = -2.09$ (diamond), and $E_{b3} = -2.19$ (triangle)] and the profiles of the BEC states are shown in the left part of Fig. 2(b). With a linear stability analysis, we confirm that all these solutions centered in $n = n_c$ are stable. This is important for two reasons. First, experimental generation of these states should be possible. Second, the scattering of small-amplitude wave packets by these modes can be viewed as a small perturbation of the latter, which will stay small if the BEC is stable.

In order to study the scattering of a propagating atom beam by the localized BEC, we consider

$$\Psi_n(t) = \phi_n(t) + b_n(t), \tag{4}$$

where ϕ_n is assumed to be "small." We linearize Eq. (1) with respect to $\phi_n(t)$, obtaining for the atom beam

$$-i\frac{\partial\phi_n}{\partial t} = (\phi_{n+1} + \phi_{n-1}) + g(2\phi_{n_c} + e^{-2iE_bt}\phi_{n_c}^*)\delta_{n,n_c}. \tag{5}$$

Because of the confined interaction region, the conditions of validity $|b_{n_c}|^2 \gg 1$ for the mean field ansatz (1) and $|\phi_{n_c}| \ll |b_{n_c}|$ for the linearization (5) are easily satisfied for large enough BECs. Far away from $n = n_c$, the solution of Eq. (5) corresponds to a propagating plane wave which satisfies (2). The localized BEC generates a scattering potential for the atom beam which has a constant and a time-dependent part. A similar equation to (5) was found in Ref. [13] for the scattering of plane waves against a discrete breather (a localized excitation on nonlinear lattices [24]). In that work, Fano resonances were studied in a full nonlinear lattice with a strongly localized nonlinear mode. This, however, implies $g \gg 1$, which makes the discrete breather a very strong scatterer for almost all incoming plane waves. Our setup allows us to tune g to smaller values and thus admits Fano resonances on a background of almost perfect transmission.

We solve the Eq. (5) by using a Bogoliubov transformation [9] given by

$$\phi_n(t) = u_ne^{-iEt} + v_n^*e^{-i(2E_b-E)t}, \tag{6}$$

and by inserting it in (5), we obtain the discrete Bogoliubov (DB) equations:

$$Eu_n = -(u_{n+1} + u_{n-1}) - g(2u_{n_c} + v_{n_c})\delta_{n,n_c}, \quad (7)$$

$$(2E_b - E)v_n = -(v_{n+1} + v_{n-1}) - g(2v_{n_c} + u_{n_c})\delta_{n,n_c}. \quad (8)$$

Here, u_n corresponds to an *open* channel which, far away from n_c, represents a propagating atom beam for which the energy is in the band $E = E_k \in [-2, 2]$. Contrary, v_n represents a *closed* channel whose extended states far away from the scattering center have $2E_b - E \notin [-2, 2]$. They are thus located outside the open channel continuum and cannot be excited in the same energy range. However, even in the absence of any coupling between both channels, the scattering center also provides a localized state in the spectrum of the closed channel with energy E_L. The localized state, by definition, is located outside the band of extended states in the v_n channel, but may be located inside the u_n-channel band $E \in [-2, 2]$. In such a case, taking the coupling between channels into account, we encounter a Fano resonance for $E_L = E_k$.

Let us consider the situation when both channels are decoupled. For this particular situation, the closed channel equation is given by

$$(2E_b - E)v_n = -(v_{n+1} + v_{n-1}) - 2gv_{n_c}\delta_{n,n_c},$$

and admits a localized solution $v_n = v_{n_c} w^{|n-n_c|}$ ($|w| < 1$), for which

$$w = -g + \sqrt{1 + g^2} \quad \text{and} \quad E = E_L \equiv 2(E_b + \sqrt{1 + g^2}).$$

We call this solution the local mode (LM). E_L corresponds to the LM energy, which is always inside the continuum of the open channel: if $g \to 0 \Rightarrow E_L \to -2$ and, if $g \to \infty \Rightarrow E_L \to 0$. Therefore, due to the time dependence of the original scattering potential, the closed channel is able to resonate with the open one at a frequency that depends on externally controllable parameters.

Keeping in mind the localized nature of the LM and the propagating one of the open channel, we make the following ansatz:

$$u_n = \begin{cases} a_1 e^{ik(n-n_c)} + b_1 e^{-ik(n-n_c)}; & n < n_c \\ c_1 e^{ik(n-n_c)}; & n \geq n_c \end{cases}, \quad (9)$$

$$v_n = \bar{v}_{n_c} \bar{w}^{|n-n_c|}. \quad (10)$$

Here, a_1, b_1, and c_1 represent the incoming, reflected, and transmitted beam amplitudes, respectively. $\bar{v}_{n_c}$ corresponds to the closed channel amplitude and $|\bar{w}| < 1$. The beam quasimomentum k can be generated in the experiment by using a phase imprinting method [25], Bragg scattering, or simply by acceleration of the matter-wave probe in an external potential. We solve analytically the scattering problem by inserting (9) and (10) in (7) and (8) for $n = n_c, n_c \pm 1$. By doing so, we obtain that the open channel satisfies (2), $a_1 + b_1 = c_1$, $\bar{w} = w$, and that the transmission is given by

$$T(k) = \frac{4\sin^2 k}{4\sin^2 k + \left(2g + \frac{g^2}{\sqrt{(E_k - 2E_b)^2 - 4} - 2g}\right)^2} \quad (11)$$

($T \equiv |c_1/a_1|^2$). Resonances occur when the denominator diverges or when $\sqrt{(E_k - 2E_b)^2 - 4} - 2g = 0$. The condition for the resonance is

$$E_k = E_L \Rightarrow k_L \equiv \arccos(-E_L/2). \quad (12)$$

Equation (12) implies that the transmission for atom beams through the BEC is reduced to zero when an LM is generated in the process, i.e., when the closed channel resonates with the open one.

In Fig. 2(b), we show the corresponding energies for the LMs, for the three values of g, $E_{L1} = -1.94$ (box), $E_{L2} = -1.84$ (diamond), and $E_{L3} = -1.69$ (triangle). The curve in that figure corresponds to Eq. (2), and the filled circles correspond to the value of k for which the resonance is expected (12). In principle, any energy in the interval $\{-2, 0\}$ can be a good candidate for the observation of a Fano resonance in this setup. But, as we will show later, the response of the system is not always the same, and it essentially depends on the BEC profile.

The transmission $T(k)$ from Eq. (11) is shown in Fig. 3 for three values of g. As g increases, the width and the position of the resonance increase. Furthermore, the more localized the BEC becomes, the stronger it reflects the atom beam off resonance. As expected from (12), increasing g (which corresponds to decreasing E_b or increasing N_b) leads to an increase of the resonance energy. By tuning the nonlinear parameter g, we can thus choose the amount of the beam which passes through the BEC. Off resonance (for larger values of k), we can select the percentage of the incoming beam that is transmitted for a defined quasimomentum. Therefore, the actual setup can be used as a 100% blockade or as a selective filter.

Now, we look for a numerical confirmation of our theoretical description. In the simulations of Eq. (1), we initialize the atom beam with a Gaussian profile:

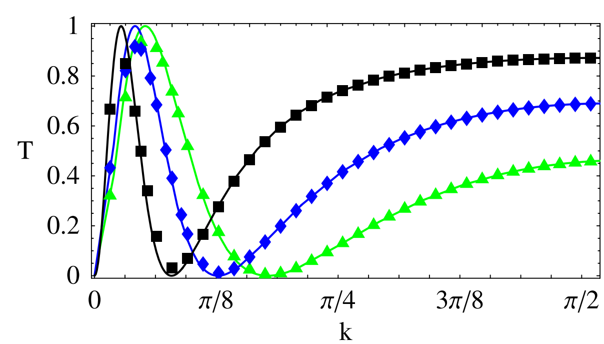

FIG. 3 (color online). Transmission T versus momentum k. Lines: Eq. (11), symbols: real time numerical simulations of Eq. (1) using wave packets for $g_1 = 0.36$ (line and boxes), $g_2 = 0.6$ (line and diamonds), and $g_3 = 0.9$ (line and triangles).

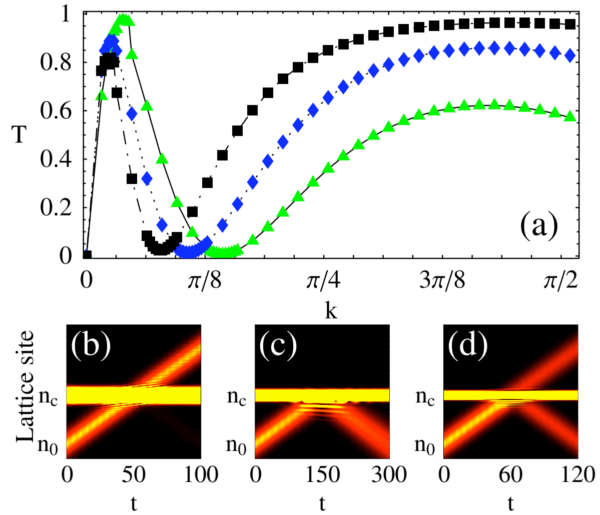

FIG. 4 (color online). Real time numerical simulations of Eq. (1) for a three-site nonlinear impurity. (a) T versus k for $g_1 = 0.36$ (boxes), $g_2 = 0.6$ (diamonds), and $g_3 = 0.9$ (triangles). Evolution of $|\Psi_n(t)|^2$: (b) g_1, $k = 1.4$, (c) g_2, $k = 0.34$, (d) g_3, $k = 0.95$. Lines are guides to the eye, only.

$\phi = \phi_0 \exp[-\alpha(n - n_0)^2]\exp[ik(n - n_0)]$ with $\phi_0 = 0.01$ and $\alpha = 0.001$. n_0 is the initial location of the center of the distribution and well separated from the BEC to avoid an initial interaction. k is the initial quasimomentum. The amplitude ϕ_0 was chosen to be very small compared to the BEC amplitude in order to justify Eq. (5). The value of α implies a spatial width of approximately 60 sites and a reciprocal width in k-space of 0.12. With this choice, we can clearly observe the resonant response of the system. In Fig. 3, the symbols denote our numerical results for g_1, g_2, and g_3. The agreement between theory and simulations is almost perfect. We have some disagreement for small values of k where the group velocity is very small and the numerical computation of T is unreliable.

Finally, we numerically study a more realistic scenario. We consider interactions to extend over three sites of the lattice as it may be realistically achieved by magnetic field variations on an atom chip: $\gamma(n) = \gamma \exp(-|n - n_c|)$ for $n = n_c$, $n_c \pm 1$ and $\gamma(n) = 0$, otherwise. By using a Newton-Raphson method, we compute the BEC profiles for g_1, g_2, and g_3. These solutions are stable with almost the same energies E_{b1}, E_{b2}, and E_{b3}. In Fig. 4(a), we present our numerical computations of T versus k. The results are similar to the ones for a single-site impurity (see Fig. 3). Figures 4(b)–4(d) show some numerical examples with a transmission of 96%, 1%, and 49%, respectively. These results show the robustness of our theoretical prediction in a more realistic experimental setup. We note that Fig. 4(d) realizes a fully coherent beam splitter for the incoming atom beam, which we have checked numerically.

In conclusion, we have investigated Fano resonances in the context of Bose-Einstein condensates in an optical lattice. The implementation of this idea can be viewed as a powerful tool for controlling the transmission of matter waves in interferometry and quantum information processes. Fano resonances rely on destructive interference and are thus inherent to wave dynamics. An observation of these resonances in atom-BEC scattering would provide, in addition to tunable filters, a new demonstration of the quantum matter-wave character of ultracold atoms.

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