Comment on "Coherent Ratchets in Driven Bose-Einstein Condensates"

Creffield and Sols (henceforth CS) [1] recently reported a finite, directed time-averaged ratchet current, for *non-interacting* quantum particles in a potential $V(x,t) = KV(x)f(t)$ with time-periodic driving $f(t) = f(t+T)$, even when time-reversal symmetry holds, as depicted with the solid line in Fig. 3 in [1]. CS chose $V(x) = \sin(x) + \alpha\sin(2x)$, $f(t) = \sin(t) + \beta\sin(2t)$ ($\beta = 0$ in their Fig. 3), and the initial condition $\Psi(x,0) = 1/\sqrt{2\pi}$. As we will explain in the following, this is incorrect; that is, time-reversal symmetry implies a vanishing ratchet current.

The asymptotic time-averaged current (TAC) is given by $J = \lim_{\tau\to\infty} J(\tau)$, where $J(\tau) = \tau^{-1}\int_0^\tau I(t)dt$. $I(t)$ is given by $I(t) = -i\int_{-\infty}^{\infty} dx \Psi^*(x,t)\frac{\partial\Psi(x,t)}{\partial x}$. Given the periodicity of the driving, $f(t) = f(t+T)$, one may analyze the evolution in terms of the system's Floquet states. The asymptotic TAC is then given by [2]

$$J = \sum_\alpha |C_\alpha|^2 \langle\langle\psi_\alpha|\hat{p}|\psi_\alpha\rangle\rangle_T = \sum_\alpha |C_\alpha|^2 \langle v_\alpha(t)\rangle_T, \quad (1)$$

where ψ_α are the Floquet eigenstates (FES), $\psi_\alpha(t+T) = \psi_\alpha(t)$, the coefficients C_α are such that $\Psi(x,0) = \sum_\alpha C_\alpha \psi_\alpha(x,0)$, $v_\alpha(t) = -i\int_{-\infty}^{\infty} dx \psi_\alpha^*(x,t)\frac{\partial\psi_\alpha(x,t)}{\partial x}$ is the instantaneous velocity of the Floquet state, and $\langle\ldots\rangle_T$ denotes the average in time over the period T. The TAC for each FES vanishes identically if $f(t_s+t) = f(t_s-t)$ for some t_s, because $v_\alpha(t_s+t) = -v_\alpha(t_s-t)$, and therefore $\langle v_\alpha(t)\rangle_T = 0$ [2]. Given that J is the weighted sum (1), it follows that $J = 0$ for $\beta = 0$ because $\sin(\pi/2+t) = \sin(\pi/2-t)$. Since the parameter K does not change the symmetries of the system, and given that the time-reversal symmetry implies a vanishing TAC, we conclude that no asymptotic directed transport occurs for any value of this parameter. CS used the stroboscopic current, $J_s(t_p,m) = \frac{1}{m+1}\sum_{n=0}^m I(t_p+nT)$. Their asymptotic stroboscopic current is given by [2]

$$J_s(t_p) = \sum_\alpha |C_\alpha|^2 v_\alpha(t_p), \quad (2)$$

where $v_\alpha(t)$ are periodic functions, $v_\alpha(t+T) = v_\alpha(t)$. Since even in the case of time-reversal symmetry instantaneous velocities are nonzero, $v_\alpha(t_p) \neq 0$, the current (2) acquires a nonzero value, which depends on the arbitrary choice of the measurement time $t_p \in [0,T)$.

Motion is a continuous process, and attempts to describe it in terms of stroboscopic characteristics only may lead to the wrong physical conclusions. The harmonic oscillator constitutes a good example: Its particle velocity is $v(t) = v_0\sin[\omega(t-t_p)]$ and, depending on t_p, the asymptotic stroboscopic averaged velocity $v_s(t_p)$ may take any value within the interval $[-v_0, v_0]$, although no directed transport occurs. We numerically verified the above conclusions

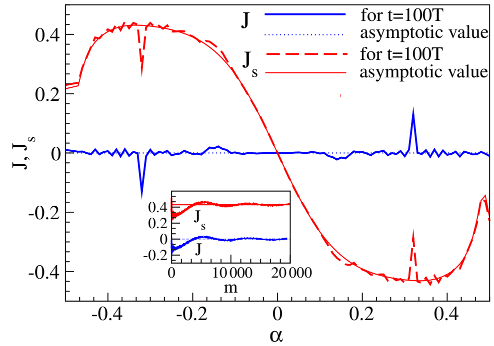

FIG. 1 (color online). $J(t)$ and the stroboscopic current $J(0,m)$ as functions of α: for $t = mT$, where $m = 100$ (thick blue solid line and thick red dashed line, correspondingly); and their asymptotic values, J, Eq. (1) (thin blue dotted line), and J_s, Eq. (2) (thin red solid line). Here $K = 2.4$ and $\beta = 0$. Inset: Dependence of $J(t = mT)$ (lower thick blue line) and $J_s(t_p = 0, m)$ (upper thick red line) on m at $\alpha = -0.32$. The thin lines are given by (1) and (2), respectively.

by performing an integration of the Schrödinger equation with the same parameters as in Fig. 3 of [1]. We used two independent methods [2,3]. The so obtained results do coincide and are depicted in our Fig. 1. For $\beta = 0$ we numerically obtain virtually zero current for all values of α, the thick (blue) solid line. The amplitudes of small fluctuations away from zero decrease systematically upon increasing the overall integration time τ; see inset in Fig. 1. These findings are therefore in full agreement with the symmetry analysis [2]. In contrast, the stroboscopic current used in Ref. [1] remains finite forever, approaching values predicted by (2). Moreover, the above symmetry analysis is not in contrast with Ref. [3], where the atom-atom interactions obey time-reversal symmetry.

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