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Review Article

Progress on artificial flat band systems: classifying, perturbing, applying

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We highlight recent progress in the study of artificial flat band systems with a threefold focus. First, we discuss single-particle flat band physics, which has advanced through the design of various flat band generators. These generators rely on the classification of flat bands in terms of compact localized states – their fundamental building blocks. A related development is the complete real-space description of flat band projectors. Next, we review studies on perturbations of flat bands, which provide new insights into the effects of disorder and, more importantly, the intricate interplay between many-body interactions and flat band physics. Finally, we survey the growing number of experimental realizations of flat bands across diverse physical platforms.

Keywords: Compact localized states; real-space projectors; many body interactions; transmon qubits; acoustic metamaterials

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1. Introduction

Periodic lattices featuring dispersionless energy bands – known as flat bands (FBs) – have become a cornerstone of modern condensed matter and wave physics. In lattices with finite-range hopping and a finite number of bands, a flat band is supported by a macroscopic number of degenerate compact localized states (CLS). Their existence originates from destructive wave interference, a

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phenomenon ubiquitous across all physical FB systems, which make CLS highly sensitive to perturbations. The appeal of FBs largely stems from the fact that a vanishing bandwidth implies zero kinetic energy, making even weak perturbations strongly nonperturbative. The ongoing search for novel and unconventional transport phases – often emerging when flat bands are perturbed – has been a major driving force behind the recent surge of research in this field.

The review by Leykam et al. [1] in *Advances in Physics: X* provided a comprehensive overview of FB research up to 2018. At that time, most studies focused on a few prototypical models, such as the Lieb lattice, the diamond chain, and the kagome lattice. These systems served as testbeds to explore the effects of perturbations (e.g. disorder or interactions) and to realize flat bands experimentally using photonic waveguides and exciton–polariton condensates [1, 2]. However, a complete and systematic algebraic classification of flat bands remained largely unexplored, despite several pioneering attempts to develop classification schemes and generator constructions. As a result, subsequent research on projectors, perturbations, and many-body interactions in flat band lattices was still in its early stages, awaiting further development.

Since 2018, the field has expanded rapidly, as reflected in Refs. [3–5]. In particular, photonics has emerged as one of the most prolific platforms for flat band studies, owing to its capability for direct structure fabrication and precise parameter control [4, 5]. Notably, this growth occurred even though flat bands were originally introduced in condensed matter physics, for example to study exotic ground states of many-body Heisenberg spin systems [6].

The present short review aims to update and extend Ref. [1] by summarizing the major advances in flat band physics achieved since 2018, with particular emphasis on (i) classification schemes, (ii) the many-body interacting regime, and (iii) experimental realizations across diverse physical platforms beyond photonics. We also discuss emerging directions and future perspectives for this rapidly evolving field.

2. Classifying, generating, projecting

Flat band lattice models arise from fine-tuning within tight-binding networks. They form continuous families of parameters in this space rather than existing as isolated points. The essential control parameters are simply the amplitudes of the compact localized states (CLS). This finite set of amplitudes defines the algebraic structure of the CLS set, determines the flat band class, and governs the resulting physical properties [5].

The classification of CLS sets is based on two criteria: orthogonality and completeness. A generic choice of CLS amplitudes typically produces a linearly independent but nonorthogonal set. Additional fine-tuning of the CLS amplitudes can render the set orthogonal. This class includes the notable case of all-bands-flat (ABF) lattices – such as the Creutz ladder – where all bands are dispersionless and transport is completely suppressed [1]. Alternatively, a different fine-tuning of the CLS amplitudes can make the set linearly dependent. This class, known as the singular flat band class [3], exists only in dimensions $d \geq 2$ and necessarily features a band touching between the flat band and one or several dispersive bands. All three classes can coexist within the same CLS controlled parametric family of a flat band lattice and can be accessed by tuning the CLS amplitudes, as illustrated in Figure 1 for the two-dimensional checkerboard lattice.

The flat band class crucially determines the lattice characteristics in both real and momentum space. In real space, the flat band projector is a key tool for analyzing lattice perturbations, and its spatial decay depends on the CLS class [7]. Orthogonal flat bands yield strictly compact projectors, whereas nonorthogonal ones produce non-compact projectors due to the overlap between

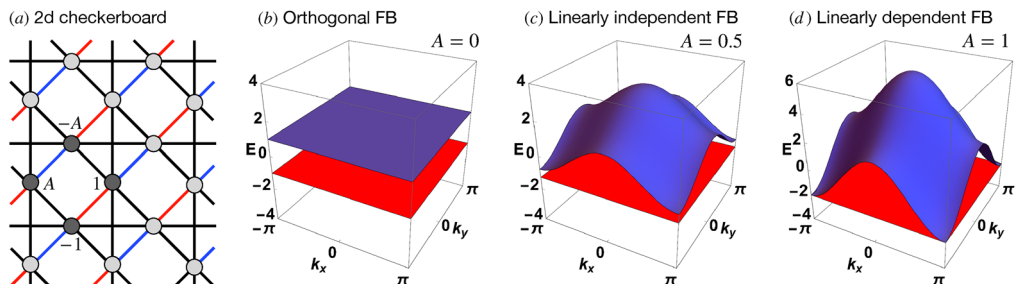


Figure 1. (a) Generalized checkerboard lattice. The hopping strengths are equal to 1 (red lines), A (black lines) and A^2 (blue lines). The CLS amplitudes are colored in black. (b) Orthogonal flat bands at $E = \pm 1$ achieved for $A = 0$. (c) Linearly independent flat band at $E = -5/4$ (red) gapped from the dispersive band (blue), achieved for $A = 0.5$. (d) Linearly dependent flat band at $E = -2$ (red) touching the dispersive band (blue), achieved for $A = 1$. This figure is inspired from some of the results presented in [7].

CLSs. The decay profile also reflects completeness: linearly independent flat bands exhibit exponential decay with algebraic prefactors, while linearly dependent (singular) flat bands display algebraic decay [7]. In momentum space, the energy of an orthogonal flat band is tunable across the spectrum. Linearly independent flat bands are necessarily gapped from all dispersive bands, whereas linearly dependent flat bands touch at least one dispersive band at one or more points. These spectral distinctions are illustrated in Figure 1(c,d).

A recent series of CLS-based flat band generators has been introduced and discussed in Ref. [5]. The special case of singular flat bands motivated the development of dedicated singular FB generators [8, 9], with particular focus on multifold band-touching points [10]. A key element common to both the study of flat band projectors [7] and singular FB generators [8–10] is the momentum-space analog of a CLS – the unnormalized Bloch compact localized state (BCLS) – and its crucial algebraic structure. As a result, elegant and simple parametric FB families can be derived and used for further studies. One fresh example is adapted from Ref. [7] and shown in Figure 1. A 2d generalized checkerboard lattice is considered, the CLS on four sites with two having fixed amplitude 1 and two having tunable amplitude $0 \leq A \leq 1$ is defined, the BCLS is computed, and the generator is applied to arrive at a tunable FB family, linearly independent for all A except for $A = 0$ (orthogonal) and $A = 1$ (singular). Thus, despite substantial progress in the understanding of flat band generation [1, 5], the field continues to evolve, producing novel tools and compelling new directions – for instance, a recently proposed Metropolis-based generator for $d = 2$ flat band lattices [11].

Even for flat bands, the Bloch eigenstates change as one moves through the Brillouin zone. These changes quantify the Quantum Geometric Tensor [12]. Its real part – the Quantum Metric – defines the *distance* between quantum FB states. One of the most profound consequences is that the superfluid stiffness – which determines the transition temperature of a superconductor – is not zero in a flat band, but bounded by the quantum metric.

3. Many body interactions

Systematic parametrizations of single-particle FB families are powerful tools for studying the impact of perturbations and for fine-tuning perturbed lattices to exhibit desired responses. Most importantly, the FB projector properties [7] provide both quantitative and qualitative insights into the action of external perturbations. Typical examples include random or correlated disorder and classical nonlinear interactions, which lead to a range of phenomena such as inverse

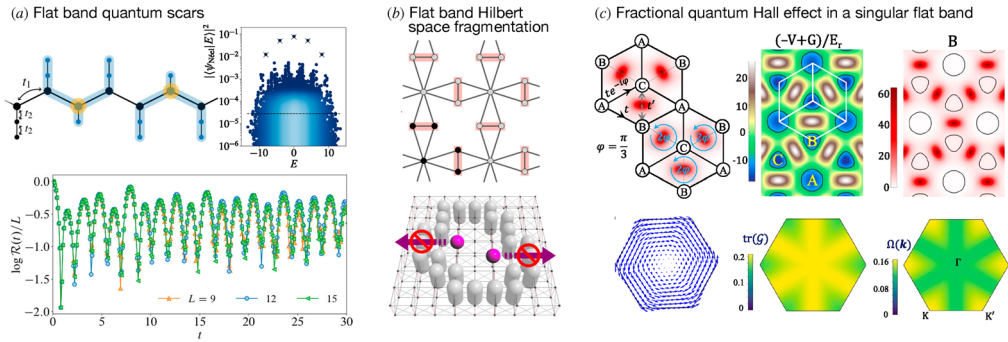


Figure 2. (a) Eight bands comb structure with flat bands $E = \pm t_2$. Projection of the eigenstates onto the Néel state, with the scars highlighted with crosses at $E = 0, \pm 2t_2, \pm 4t_2$. Return probability \mathcal{R} of a Néel state excitation. Reprinted with permission from Ref. [15]. (b) Two examples of many-body flat band lattices supporting Hilbert space fragmentation for interacting bosons (top) and hardcore bosons (bottom). Reprinted with permission from Refs. [19, 20]. (c) Top: Dice lattice with pseudomagnetic field (red color); potential landscape and pseudomagnetic field for a possible implementation. Bottom: Berry connection; Berry curvature; trace of the quantum metric tensor of the singular flat band. Reprinted with permission from Ref. [21].

Anderson (metal–insulator) transitions, Bloch oscillations induced by electric fields, and compact breathers and caging effects arising from Kerr nonlinearities – topics covered in detail for photonic applications in Ref. [5].

Single-particle CLSs historically served as building blocks for exotic many-body states in interacting quantum systems, such as Heisenberg spin lattices [6]. Their construction allows for new insights into the thermodynamic properties of quantum Heisenberg antiferromagnets in frustrated geometries, for instance the kagome bilayer [13]. Recent developments include both theoretical and experimental studies of entire new classes of quantum materials, collectively referred to as kagome metals [14].

Non-equilibrium quantum phenomena likewise emerge from many-body interactions in FB lattices. Two prominent examples are *quantum many-body scars* and *Hilbert-space fragmentation*. Quantum scars are non-thermalizing many-body states characterized by low entanglement, which have been observed in several interacting FB lattices [15–18]. For example, the system of interacting fermions illustrated in Figure 2(a) exhibits scars at five distinct energies. When initialized in an antiferromagnetic Néel state (in this case, one particle on every third orange site along the horizontal subchain of the lattice), the return probability shows coherent oscillations rather than rapidly decay to zero [15].

If FBs emerge from fine-tuning the single-particle sector, one may ask: why not fine-tune the interaction sector as well to obtain new, exotic quantum phases? Such many-body fine-tuning can yield an extensive set of local commuting operators that fragment the Hilbert space into disconnected subsectors [19, 20, 22, 23]. This mechanism can completely suppress charge transport along the lattice – a phenomenon known as *many-body flat band localization* [19, 20, 22]. Two representative systems exhibiting Hilbert-space fragmentation are shown in Figure 2(b). In the upper case, the fragmentation arises from finetuned ABF single particle lattices with carefully designed interaction between neighboring sites indicated by colored blocks [19, 22]; in the lower case, the shattering of the Hilbert space results from the hard-core boson limit (infinite interaction strength) on a $2d$ cross stitch single particle model with one flat and one dispersive band. The figure shows a loop excitation of many CLSs which act as an impenetrable fence for additional hard core bosons trapped inside [20].

Further research spans across traditional domains ranging from superconductivity in FB lattices [24, 25] to paired transport of interacting bosons in Aharonov–Bohm cages [26] and the anomalous fractional quantum Hall effect in a singular flat band [21]. The latter example is illustrated in Figure 2(c), showing the Dice lattice dressed by a magnetic field. This model, realizable via a carefully engineered potential landscape, hosts a singular flat band with non-trivial Berry connection, curvature, and quantum metric tensor, which together give rise to the fractional quantum Hall effect.

4. Experimental applications

Arrays of optical waveguides have long been considered the optimal platform for experimentally investigating FB phenomena. Indeed, the precision offered by femtosecond laser writing allows for the precise engineering of an ever-growing number of more and more complex FB lattices, also including various types of perturbations, e.g. external fields, periodic driving, interactions [2, 4, 5]. Nevertheless, significant technological progress has recently enabled other experimental platforms to emerge and stand at the forefront of FB research.

A fresh example of this is the realization of CLS in electric FB lattices formed by inductors and capacitors. In one dimension, these electric circuits can be arranged to form orthogonal FBs (the diamond lattice) and linearly independent FBs (the stub lattice), whose CLS can be continued as exact solutions in the nonlinear regime [27]. Notably, this approach is highly versatile: the circuits in the diamond chain can be reconfigured to emulate the π -flux within each plaquette, hence yielding the Aharonov-Bohm caging effect [28]. Moreover, the approach is scalable for generating $2d$ lattices that, for example, (i) feature boundary flat bands with topological spin textures and quantized π -Berry phases [29] and (ii) emulate interaction-induced flat-band localization phenomena of two correlated bosons in $1d$ Aharonov-Bohm cages [30].

Acoustic systems also offer a compelling platform for FBs. Examples include the experimental realization of CLSs in singular FB lattices inscribed in acoustic plates [31], and of hexagonal FB lattices created in phononic metamaterials by carefully assembling disks with magnetic couplings [32]. Notably, in the latter example, the magnetic couplings can also be additionally fine-tuned to render all bands flat. Furthermore, acoustic systems offer ways to realize topological phenomena – e.g. the non-Abelian Thouless pumping in FB systems which was originally proposed in Ref. [33] for photonic lattices. As shown in Figure 3(a), non-Abelian pumping can be realized using an array of modulated acoustic waveguides arranged as a generalized one-dimensional Lieb lattice featuring two zero-energy FBs [34]. The typical displacing and shuffling the CLSs throughout the network, a hallmark of this phenomenon, is shown both numerically (bottom left) and experimentally (bottom right).

As remarked by these experimental results as well as by those in the previous sections, ABF lattices attract significant interest. Indeed, their fully quenched kinetic energy enables exotic phenomena that go beyond traditional FBs. For instance, these systems exhibit the complete absence of single particle transport. However, this effect reversed due to Hubbard interactions, which induce two-particle paired transport [1, 16, 26]. Such a breakdown of full halt of transport has been recently verified using Rydberg atoms in optical arrays [36], with a synthetic lattice of ultracold ^{87}Rb atoms [37], and with superconducting transmon qubits [35]. Figure 3(b) shows the latter case by displaying on the left the π -flux rhombic plaquette of qubits, which is obtained by properly orienting one of the four qubits. The top-right panel demonstrates the single particle caging, where a particle starting from the left qubit over time never occupies the right one. The bottom-right panel instead shows the caging breakdown for two interacting photons.

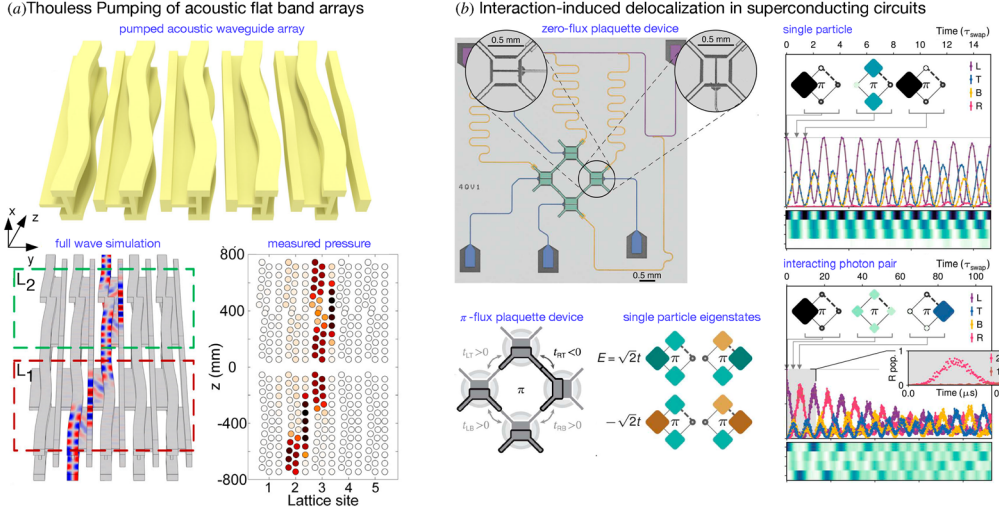


Figure 3. (a) Acoustic platform. Top: configurations of the acoustic waveguide arrays driven via a pumping loop. Bottom: simulated sound wave propagation (left) and measured pressure at discrete locations along the waveguides. Reprinted with permission from Ref. [34]. (b) Superconducting transmon qubit platform. Left top: zero-flux plaquette, with zoomed qubit orientations that yield zero (left) and π -flux (right) plaquette. Left bottom: detailed hopping profile of the π -flux plaquette (left) and single particle eigenbasis (right). Right: average qubit populations in the different qubits of the plaquette as function of time for single photon particle (top) and two interacting photon particles (bottom). Reprinted with permission from Ref. [35].

These experiments indicate the rapid development of the field of flat bands towards practical technologies, beyond foundational studies. Flat bands have been recently used for the realization of lasing using silicon waveguide-integrated metasurfaces [38] and ultra-compact cavities [39]. Another exciting direction is the observation of topological flat bands nanosheets using photoelectric detections, a result which opens venues for new material platforms for organic photodetectors [40]. The field has also seen an avalanche of novel results on kagome metals [41–43], which include unusual magnetism [44], high-order topological phases [45] and coherent charge transport phenomena [46]. These findings remark this as an intriguing area for the design of unusual quantum states and novel phenomena, promising far-reaching applications in material science. We refer the readers to the following review [47] for more insights.

5. Conclusions and outlook

Over the past decade, the field of flat band (FB) physics has matured from a collection of isolated model studies into a coherent and rapidly expanding research frontier that bridges condensed matter, photonics, acoustics, electric circuits, and quantum technologies. The recognition that compact localized states (CLS) form the universal backbone of all flat band systems has enabled a unified algebraic framework that links lattice geometry, interference patterns, and spectral properties. This paradigm shift – from analyzing specific lattices toward understanding the algebraic structure and classification of CLS families – has opened a systematic route for constructing, manipulating, and fine-tuning flat bands across dimensions and physical platforms.

The classification of FBs into orthogonal, linearly independent, and singular families, together with the associated projector formalism, now provides a clear correspondence between real- and momentum-space features. This structural understanding has demystified phenomena such as

algebraic versus exponential localization, band touching, and tunable degeneracies. Moreover, the development of analytical and numerical flat band generators has transformed FB design from an art into an algorithmic process. These tools not only simplify the exploration of parameter spaces but also serve as essential stepping stones for investigating perturbations, nonlinearities, and disorder. The emergence of machine-inspired approaches, such as Metropolis-type algorithms, signals the beginning of automated flat band discovery across higher-dimensional and topologically enriched systems.

Equally transformative has been the exploration of many-body effects in flat band lattices. By suppressing kinetic energy, FBs amplify the role of interactions, giving rise to rich nonperturbative phenomena: quantum scars, Hilbert-space fragmentation, many-body localization without disorder, and interaction-induced transport in otherwise caged systems. The conceptual symmetry between single-particle fine-tuning and many-body fine-tuning illustrates how FBs serve as fertile ground for engineering exotic quantum phases. These advances connect to fundamental questions about ergodicity, thermalization, and the emergence of nontrivial correlations in constrained quantum systems – linking FB physics to the broader landscape of nonequilibrium quantum dynamics.

On the experimental front, remarkable progress has expanded the scope of accessible FB platforms. Photonic waveguide arrays remain a benchmark for precision control and visualization of CLS dynamics, yet complementary platforms – acoustic metamaterials, electric circuits, superconducting qubits, and ultracold atoms – have demonstrated equivalent or even superior tunability. Experiments now routinely probe interaction-driven delocalization, topological pumping, and fractional quantum Hall effects within flat or singular bands. The convergence of these diverse implementations establishes FBs as a universal motif rather than a platform-specific curiosity. Importantly, the cross-fertilization between theory and experiment has accelerated the pace of discovery, revealing FB signatures even in correlated electron materials such as kagome metals.

Looking ahead, flat band research is poised to enter a new integrative phase. Future directions include the design of higher-order topological FBs, controlled inclusion of spin-orbit coupling, and the exploration of nonequilibrium and driven-dissipative regimes where flatness competes with gain and loss. The marriage of algebraic design principles with experimental scalability promises not only deeper theoretical understanding but also tangible applications – from robust photonic circuitry to designer quantum matter with controllable interactions. As a unifying framework for localization, topology, and correlation, flat band physics continues to exemplify how elegant mathematical structure can guide and inspire the search for novel physical realities.

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